The Open University

MU120
Open Mathematics

Preparatory Resource
Book B

Modules 5–7

The Open University

MU120

Open Mathematics

Preparatory Resource Book B

Modules 5–7

Acknowledgements

Grateful acknowledgement is made to the following sources for permission to reproduce material in this book.

Tables on p.10: The Post Office, *All You Need To Know*, Leaflet, March 2000; Example 4 on p.13: Extract from *A Guide to Main Line Services, 24 September 2000 to 19 May 2001*, Midland Main Line Limited; Example 5 on p.14: Unijet, *Sunbusters Brochure*, Summer 2000, Issue 1; Table on p.15: Office for National Statistics, *Social Trends 29*, 1999, Crown copyright material is reproduced under Class Licence Number C01W0000065 with the permission of the Controller of HMSO and the Queen's Printer for Scotland; Figure on p.23: *Which?*, published by Consumers' Association, 2 Marylebone Road, London, NW1 4DF (for further information, please telephone 0800 252 100).

Every effort has been made to trace all copyright owners, but if any has been inadvertently overlooked, the publishers will be pleased to make the necessary arrangements at the first opportunity.

The Open University, Walton Hall, Milton Keynes, MK7 6AA.

First published 1995. Second edition 2001. Third edition 2008.

Edited, designed and typeset by The Open University, using the Open University TEX System.

Printed and bound in the United Kingdom by The Charlesworth Group, Wakefield.

SUP 00252 2

3.1

Contents

How to use this book

Read this first
This is a resource book. It has been written for you to refer to when you need to revise topics. Use the material in conjunction with the unit *Preparing for Open Mathematics*, which gives advice as to the modules that you may find most helpful to study in relation to particular topics.

This is the second of two preparatory resource books. Working gradually through the material in these books will build up your confidence in your basic skills, ready for the start of Open Mathematics.

The first book, *Preparatory Resource Book A*, contains four modules, and this second book has three modules covering the topics 'Diagrams, charts and graphs', 'Language, notation and formulas', and 'Geometry'.

The modules are divided into two or three sections, and each section has the following structure.

First there are some diagnostic questions headed 'Try these first'. Each question relates to a certain topic, so it is advisable to attempt all of the questions in order to get an idea as where to focus your efforts. The answers are given after the questions; alongside each answer is a note in the margin indicating which part of this resource book you should study in more detail if you found the question on a particular topic difficult and want to do some follow-up work.

Then come various subsections, each focusing on a topic identified in the diagnostic questions. You need not study a subsection in detail if you feel reasonably confident about the topic. Each subsection explains the topic and has a number of worked examples.

At the end of each subsection there are exercises headed 'Try some yourself'. These give you practice in applying what you have just learned or revised, and include opportunities for putting ideas into words and reflecting on your learning. Aim to do these exercises as soon as you have studied the relevant subsection. You can check your solutions against those given at the end of the book.

Some of the examples and exercises in this book require the use of a calculator. They are marked with a calculator icon in the margin. Instructions covering all the calculator techniques you need here are given in Chapter 1 of the *Calculator Book*. You may wish to defer doing the calculator exercises in this book until you have reached a suitable point in the *Calculator Book*, then you can come back to them.

Remember you can return to this resource book at any time during your study of MU120 if you find you need to revise something.

Module 5 Diagrams, charts and graphs

This module has two aims: firstly, to help you read and interpret information in the form of diagrams, charts and graphs, and secondly, to give you practice in producing such diagrams yourself.

The first section deals with interpreting and drawing diagrams to a particular scale. The next section, Section 5.2, helps you to extract information from tables and charts. The final section deals with drawing graphs using coordinate axes—a very important mathematical technique that is used throughout MU120.

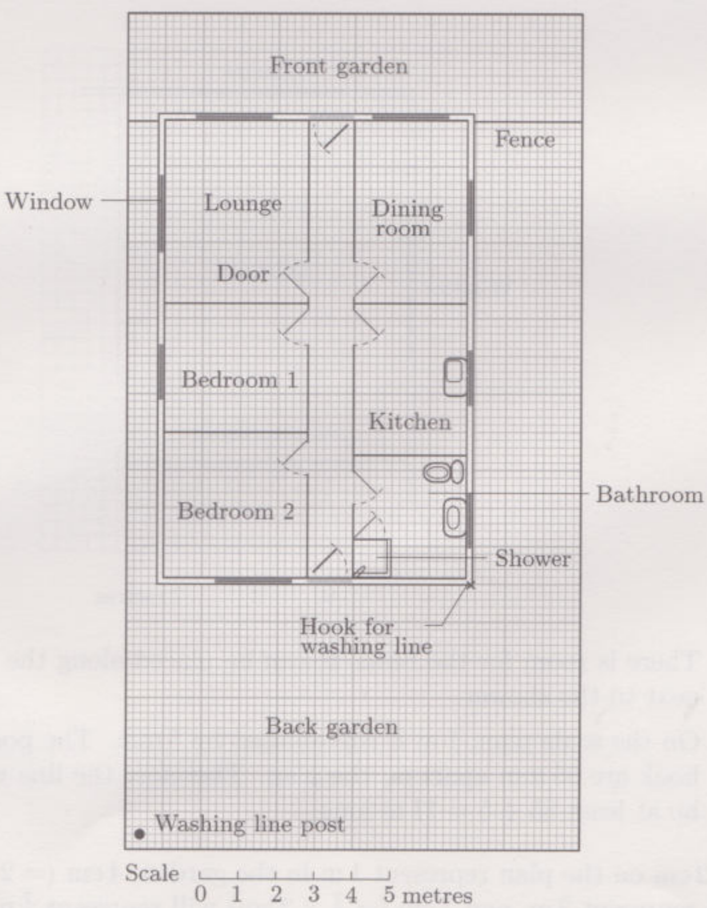
5.1 Scale diagrams

In this section you will need to make accurate measurements with a ruler.

Try these first

These are diagnostic questions. Try them to see which topics you need to revise.

1 Below is a scale plan of a new bungalow and its garden.



(a) How wide is the back garden?

(b) What are the dimensions of the lounge?

(c) Wall units (of full room height) come in sections 1 m wide and 0.3 m deep. How many units can be fitted round the walls of the

lounge without blocking the windows or door? Draw a larger scale plan of the lounge to show how these wall units can be arranged.

(d) Is the bathroom big enough for a 1.8 m by 0.8 m bath?

(e) A washing line is to be run from the washing line post to the hook on the corner of the house outside the bathroom. What is the minimum length that the line must be?

2 A scale plan of a garden is drawn using a scale in which 2 cm represent 1 m. The lawn is represented on the plan by a 4 cm by 7 cm rectangle. What is the actual size of the lawn?

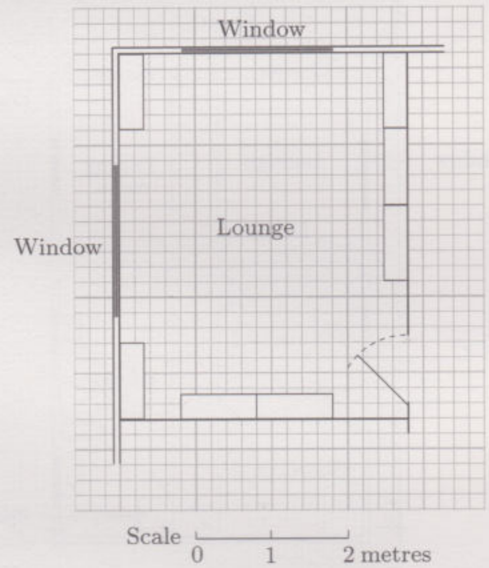
Check your answers

Follow-up in Section 5.1.

1 (a) The side of 1 big square on the plan represents 2 m. The garden is 6 big squares wide on the plan. This means it is actually 12 m wide.

(b) The lounge is 19 small squares by 24 small squares on the plan. Each small square represents 0.2 m. So the lounge is 3.8 m by 4.8 m.

(c) There is room for a maximum of seven wall units: one on either side of the side window; none on the front wall; three on the wall next to the hall; and two on the wall adjoining bedroom 1. One possible arrangement is shown below.



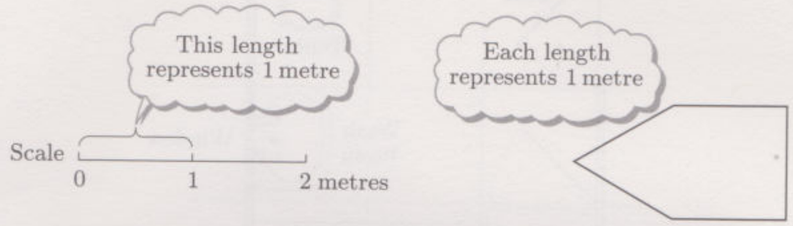
(d) There is room for the bath. It can be placed along the back wall, next to the shower.

(e) On the scale plan, 1 m is represented by 5 mm. The post and the hook are 55 mm apart on the plan. Therefore the line will have to be at least $55 \div 5 = 11$ m long.

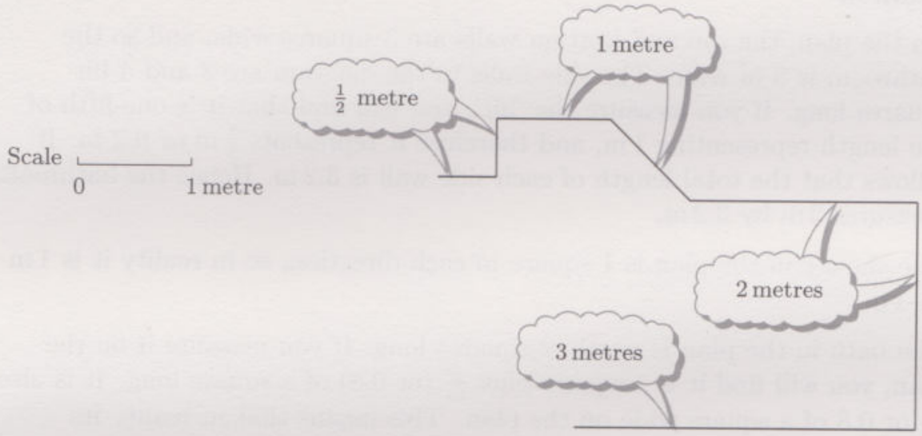
Follow-up in Section 5.1.

2 As 2 cm on the plan represent 1 m in the garden, 4 cm ($= 2 \times 2$ cm) will represent 2 m, and 7 cm ($= \frac{7}{2} \times 2$ cm) will represent $\frac{7}{2}$ m or 3.5 m. Hence the lawn is 2 m by 3.5 m.

Plans of houses and instructions for assembling shelves, etc. often come in the form of **scale diagrams**. Each length on the diagram represents a length relating to the real house, the real shelves, etc. Often a scale is given on the diagram so that you can see which length on the diagram represents a standard length, such as a metre, on the real object. This length always represents the *same* standard length, wherever it is on the diagram and in whatever direction.



Other lengths may represent fractions or multiples of this standard length. Thus, lengths which are half as long on the diagram represent lengths which are half as long in reality; lengths which are twice as long on the diagram represent lengths which are twice as long in reality; and so on.

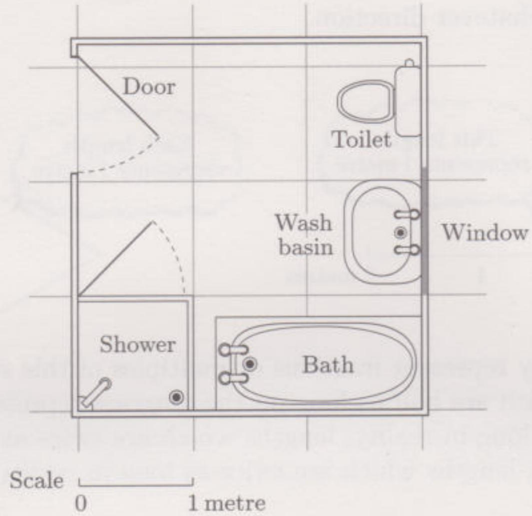


Scale diagrams are often drawn on a square grid, as in the plan of the bungalow on page 5. It is then possible to count squares on the grid rather than measure lengths on the diagram. Care must be taken with either method: the ends of a length may fall between the marks on the ruler, or the grid lines may not be equally spaced.

Example 1

Below is a scale plan of a bathroom. What are the dimensions of the bathroom, the shower and the bath? What is the width of the bathroom door?

The background squares show the length representing 1 m.



Solution

On the plan, the top and bottom walls are 3 squares wide, and so the bathroom is 3 m wide. The side walls in the diagram are 3 and a bit squares long. If you measure the 'bit', you will find that it is one-fifth of the length representing 1 m, and therefore it represents $\frac{1}{5}$ m or 0.2 m. It follows that the total length of each side wall is 3.2 m. Hence the bathroom measures 3 m by 3.2 m.

The shower in the plan is 1 square in each direction, so in reality it is 1 m by 1 m.

The bath in the plan is nearly 2 squares long. If you measure it on the plan, you will find it is 1 square plus $\frac{8}{10}$ (or 0.8) of a square long. It is also $\frac{8}{10}$ or 0.8 of a square wide on the plan. This means that in reality its dimensions are 1.8 m by 0.8 m.

As the doorframe is 1 square wide on the plan, the actual door is 1 m wide.

Example 2

(a) The scale on a diagram is such that 2 cm represent 1 m. What lengths do 6 cm, 0.2 cm, 3 cm, 3.6 cm and 0.5 cm represent?

(b) A window is 2.3 m wide and 1.4 m high. Draw a scale diagram of the window, using a scale in which 2 cm represent 1 m.

Solution

(a) Because you are being asked to convert lengths on the diagram into real lengths, it is easiest to work with a diagram length of 1 cm. As 2 cm represent 1 m, 1 cm will represent 0.5 m. Then

6 cm represent $0.5 \times 6\,\text{m} = 3\,\text{m}$,

0.2 cm represents $0.5 \times 0.2\,\text{m} = 0.1\,\text{m}$,

3 cm represent $0.5 \times 3\,\text{m} = 1.5\,\text{m}$,

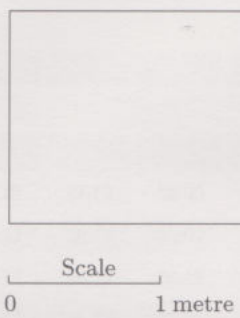
3.6 cm represent $0.5 \times 3.6\,\text{m} = 1.8\,\text{m}$,

0.5 cm represents $0.5 \times 0.5\,\text{m} = 0.25\,\text{m}$.

(b) Here 1 m in reality is represented by 2 cm on the diagram. So

2.3 m are represented by $2.3 \times 2\,\text{cm} = 4.6\,\text{cm}$,
1.4 m are represented by $1.4 \times 2\,\text{cm} = 2.8\,\text{cm}$.

Therefore, the scale diagram of the window should look like this:



Try some yourself (5.1)

Solutions on page 104.

1 On the plan of the bathroom in Example 1, what is the width of the window and what are the dimensions of the wash basin?

2 On a scale diagram, 5 cm represent 1 m. What lengths do the following represent: 10 cm, 20 cm, 1 cm?

3 On a map of a new town, 2 cm represent 1 km. What lengths on the map represent the distances of 10 km, 5 km and 0.5 km in the town?

4 Draw a scale plan of the garden described below, using a scale in which 0.5 cm represents 1 m.

The garden is rectangular and measures 10 m by 20 m. It has flowerbeds that are 2 m wide along the whole of one of the long sides and along both of the short sides. A 1.5 m wide path occupies the rest of the other long side. Another path, also 1.5 m in width, makes a T-junction with this path and leads straight to a sundial at the centre of the garden.

5.2 Tables and charts

Try these first

These are diagnostic questions. Try them to see which topics you need to revise.

1 The tables overleaf give postage rates from the UK to overseas destinations in 2000. How much would it cost to send a small packet weighing 135 g to Spain by airmail? By surface mail?

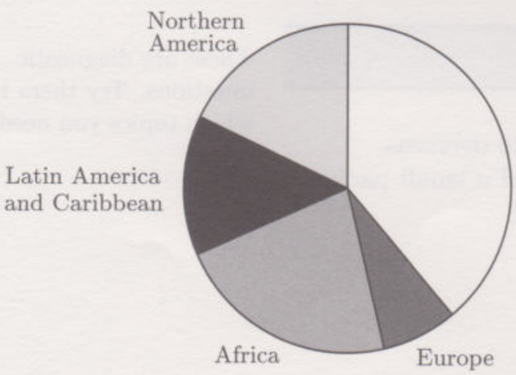
Airmail

	Letters			Small Packets			Printed Papers		
weight not over	Europe	World Zone 1	World Zone 2	Europe	World Zone 1	World Zone 2	Europe	World Zone 1	World Zone 2
Postcards	**£0.36**	**£0.40**	**£0.40**						
10g		£0.45	£0.45						
20g	£0.36	£0.65	£0.65						
40g	£0.50	£1.00	£1.07						
60g	£0.65	£1.35	£1.49						
80g	£0.80	£1.70	£1.91						
100g	£0.95	£2.05	£2.33	£0.87	£1.15	£1.15	£0.75	£1.15	£1.15
120g	£1.10	£2.40	£2.75	£0.96	£1.32	£1.34	£0.83	£1.32	£1.34
140g	£1.25	£2.75	£3.17	£1.05	£1.49	£1.53	£0.91	£1.49	£1.53
160g	£1.40	£3.10	£3.59	£1.14	£1.66	£1.72	£0.99	£1.66	£1.72
180g	£1.55	£3.45	£4.01	£1.23	£1.83	£1.91	£1.07	£1.83	£1.91
200g	£1.70	£3.80	£4.43	£1.32	£2.00	£2.10	£1.15	£2.00	£2.10

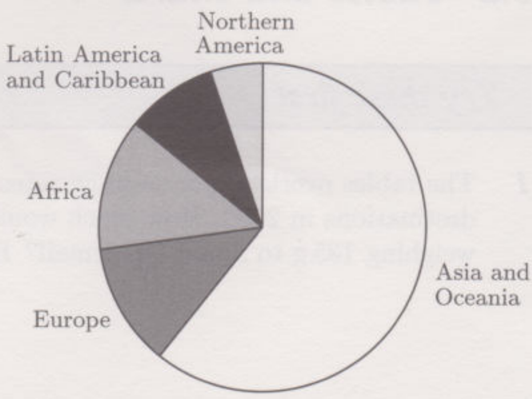
Surface mail

	Letters	Small Packets & Printed Papers
weight not over	World Zone 1 & 2	All destinations
Postcards	**£0.36**	
20g	£0.36	
60g	£0.58	
100g	£0.83	£0.57
150g	£1.16	£0.76
200g	£1.49	£0.95
250g	£1.82	£1.14
300g	£2.15	£1.33
350g	£2.48	£1.52
400g	£2.81	£1.71
450g	£3.14	£1.90

2 These pie charts represent the proportions of the world's land area and population for various regions at the end of the twentieth century.



Land area



World population

Make rough estimates of the fraction of the land area and the fraction of the population that are in each of the following regions:

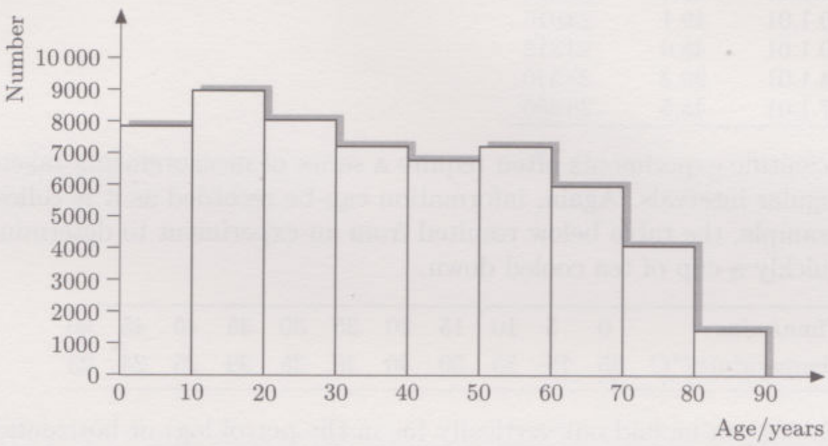
(a) North America (b) Asia and Oceania (c) Europe

(d) Africa.

3 The frequency diagram below shows the numbers of people in different age groups in a sample of the UK population.

(a) What is the width of each age group?

(b) Which age group contains the largest number of people?



Check your answers

1 Airmail £1.05. Surface mail £0.76. Follow-up in Section 5.2.1.

2 (a) North America has approximately a sixth of the land area and about a twentieth of the population. Follow-up in Section 5.2.2.

(b) Asia and Oceania take up a little more than one third of the land area and have almost two thirds of the population.

(c) Europe has approximately a twelfth of the land area and an eighth of the population.

(d) Africa has a quarter of the land area and just over an eighth of the population.

(Note: Your answers may differ slightly as these are only rough estimates.)

3 (a) The width of each age group is 10 years. For example, the ten ages 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 years make up the first age group. Follow-up in Section 5.2.3.

(b) The largest number of people are in the age group 10–19 years.

5.2.1 Tables

In everyday usage, you will commonly see the word 'data' used either as a plural noun ('The data are gathered by questionnaire') or as a singular noun ('Data is stored on the computer'). In MU120, 'data' is used as a plural noun.

Experiments or surveys usually generate a lot of information from which it is possible to draw conclusions. Such information is called *data*. Data are often presented in newspapers or books.

One convenient way to present data is in a **table**. Sometimes information can be recorded in a table as it is collected. For instance, a motorist may keep a log of a car's petrol consumption by recording the relevant information each time the car is filled with petrol:

Date	Litres	Mileage
4.1.01	49.4	22 695
10.1.01	49.1	23 016
20.1.01	48.6	23 312
24.1.01	29.2	23 540
27.1.01	48.5	23 950

Scientific experiments often require a series of measurements taken at regular intervals. Again, information can be recorded as it is collected. For example, the table below resulted from an experiment to determine how quickly a cup of tea cooled down.

Time/mins	0	5	10	15	20	25	30	35	40	45	50
Temperature/°C	85	78	55	50	46	40	35	30	25	24	23

Columns are vertical; rows are horizontal.

Tables can be laid out vertically (as in the petrol log) or horizontally (as in the tea experiment). Each column or row heading should indicate what is being measured and the unit of measurement.

A table is not merely a convenient way of presenting data. It can often facilitate comparisons and can lead to conclusions that would have been difficult to deduce from the separate data, as the next example shows.

Example 3

Sasha travels to work by car. The table indicates her journey times over a two-week period.

		Mon	Tue	Wed	Thu	Fri
Journey time/mins	Week 1	14	16	20	13	15
	Week 2	15	17	14	10	17

(a) What were her journey times on Thursday in week 1 and on Monday in week 2?

(b) What was the shortest journey time, and when did it occur?

(c) Suggest reasons for such a wide range of journey times.

Solution

(a) The journey times were 13 minutes and 15 minutes.

(b) Reading across both rows of the table, the lowest number is 10. So the shortest journey time was 10 minutes on Thursday in week 2.

Although the variations are only a few minutes, these variations are sizeable percentages of the total journey times which are short.

(c) Possible reasons for the variations in Sasha's journey times could include: traffic jams, travelling by different routes, going at different times of the day (her working hours may not be regular).

It is important to appreciate that, although you can *state* factual conclusions, you can often only *suggest* reasons. In many cases, interpretation of data depends on your own experience or on some other information not included in the table.

Example 4

Look carefully at the table below, which is one section of the Midland Mainline train timetable.

Yorkshire → East Midlands → London

From 7 January to 25 March 2001 **Sundays**

40

			TS ⊗		TS ⊗			TS ⊗	
York	8 d								1702e
Leeds	10 d			1506e					1730e
Wakefield Westgate	7 d			1522e					1742e
Doncaster	7 d				1520e		1621e		1721e
Barnsley	d				1515e		1609e		1715e
Meadowhall	d			1607e	1535e				1741e
Sheffield	7 d			1625	1609		1720		1820
Chesterfield	d			1638	1622		1733		1833
Matlock	d								
Matlock Bath	d								
Cromford	d								
Whatstandwell	d								
Ambergate	d								
Belper	d			1614f					1758f
Duffield	d								
Burton-on-Trent	d		1600						
Willington	d		1606						
Derby	10 d		1623	1703			1758		1859
Long Eaton	d		1632			1705g	1809	1752g	1803f
Alfreton	d				1614g				
Langley Mill	d								
Nottingham	8 d	1634			1655	1729	1747h	1836	
Beeston	d	1614g		1652k	1701	1736	1736h	1805g	1844h
Loughborough ✈	d		1641	1705k	1712	1748	1817		1915
Leicester	5 a	1700	1654	1728	1725	1800	1830	1859	1928
Leicester	5 d	1700 ↙	1705	1730	1735	1800	1830	1900	1930
Market Harborough	d		1720		1749		1846	1915	
Corby ▬	d		1700		1731		1826	1856	1922
Kettering	d		1731		1800		1855	1926	1952
Wellingborough	d		1739		1807		1904	1934	
Bedford	7 a		1754				1919		
Luton	10 a		1849c	1822		1852	2019c	2001	2022
Luton Airport Parkway ✈	a		1812	1852b		1922b	2022c	2022b	2042b
Gatwick Airport ✈	a		2016c	2018b		2048b	2148c	2148b	2218b
London St. Pancras	a	1822	1848	1855	1904	1925	2005	2035	2056

Key to timetable symbols

- **a** Arrival time.
- **b** Change at Luton.
- **c** Change at Bedford.
- **d** Departure time.
- **e** Change at Sheffield.
- **f** Change at Derby.
- **g** Change at Nottingham.
- **h** Change at Loughborough.
- **k** Change at Leicester.

▬ Corby to Kettering rail-bus link.

✈ Connections to International Airports, see pages 10 and 11 for details.

⊗ This train is non-smoking.

8 Minimum time needed to change trains, in minutes.

↙ Interchange between these trains is possible for customers with First Open and Standard Open tickets. This will allow an earlier arrival time in London. Customers with First Saver, Saver and Supersaver tickets may also change trains, if their tickets are valid on both services.

TS Turbostar Train shown in green. First Premier is not available on Turbostar Trains. These trains are all non-smoking.

Please note:
SuperSaver tickets are not valid for travel on Fridays or on certain other peak days, please ask for details.

Only some of the following questions can be answered from the part of the timetable on the preceding page. Where possible, answer the question; otherwise say why it is not possible to give an answer.

(a) How long does the journey take from Sheffield to Leicester on the 16:09 and 16:25 trains?

(b) If you arrive at Sheffield at 5 pm on a Saturday afternoon, when would you reach London?

(c) To get to Luton Airport by 6 pm, when should you leave Leicester?

(d) Describe the journey from Belper to Kettering, starting from Belper as soon after 4 pm as possible.

Solution

(a) The 16:09 train from Sheffield arrives in Leicester at 17:25. The journey time is 1 hour 16 minutes.

The 16:25 train arrives at 17:28. The journey time is 1 hour 3 minutes.

(b) The timetable given is for *Sunday*, so this question cannot be answered.

(c) The timetable shows that there is a train which reaches Luton Airport at 18:12, leaving Leicester at 17:05. However, if it is necessary to reach the airport *by* 6 pm (18:00) you would need to look at the previous page of the timetable, and this has not been supplied here.

(d) Leave Belper at 16:14 and change at Derby. Catch the 17:03 from Derby to Leicester, arriving at 17:28. Catch the Turbostar train from Leicester at 17:35, arriving in Kettering at 18:00.

Example 5

This table is taken from a brochure for holidays in Turkey. Use it to answer the questions that follow.

	MARMARIS/ICMELER S/C SELECTION				ICMELER HOTEL SELECTION		HISARONU HOTEL SELECTION		ALL-INCLUSIVE SELECTION		CHILD PRICES 7 & 14 NIGHTS
HOLIDAY CODE	233				234		239		232		
ACCOMMODATION TYPE	STUDIO		1 BEDROOM		TWIN		TWIN		TWIN/DOUBLE		
BOARD BASIS	SELF-CATERING		SELF-CATERING		BED & BREAKFAST		BED & BREAKFAST		ALL-INCLUSIVE		
NO. ADULTS SHARING	2/3		3/4		2		2		2		
NO. OF NIGHTS	7	14	7	14	7	14	7	14	7	14	
DEPART UK BETWEEN	ADULT	ADULT	ADULT	ADULT	ADULT	ADULT	ADULT	ADULT	ADULT	ADULT	1ST CHILD
01 MAY–10 MAY '00	245	255	215	225	245	265	249	269	379	535	45
11 MAY–17 MAY	259	309	229	269	259	285	265	289	395	549	
18 MAY–24 MAY	299	389	269	329	275	339	279	345	405	649	125
25 MAY–31 MAY	339	395	299	329	299	345	305	349	485	679	
01 JUN–14 JUN	289	329	259	289	275	309	279	315	445	629	89
15 JUN–21 JUN	299	349	269	309	285	329	289	335	449	655	
22 JUN–05 JUL	315	379	275	329	305	349	309	355	469	695	95
06 JUL–16 JUL	339	415	315	365	309	365	315	369	485	715	
17 JUL–09 AUG	369	455	355	395	345	395	349	399	525	745	125
10 AUG–16 AUG	365	455	349	389	349	399	355	409	539	755	
17 AUG–23 AUG	355	445	339	375	339	389	345	395	525	725	105
24 AUG–30 AUG	349	399	309	339	329	385	335	389	489	669	
31 AUG–06 SEP	319	369	305	315	305	349	309	355	459	645	
07 SEP–13 SEP	309	359	285	305	299	345	305	349	455	639	
14 SEP–20 SEP	295	355	275	295	289	339	295	345	449	635	
21 SEP–27 SEP	285	305	265	275	279	309	285	315	435	585	65
28 SEP–31 OCT	265	279	239	245	245	275	249	279	395	559	69

SUPPLEMENT SEASON	ADULTS	LOW	MID	HIGH	LOW	MID	HIGH
	3	-	-	-	1.30	2.60	4.55
SUPPLEMENTS PER PERSON PER NIGHT	2	2.25	4.45	7.10	3.95	7.80	11.70
	1	8.95	17.70	24.40	11.80	23.35	32.15

Prices are in £'s per person based on adult occupancy shown. All prices should be read in conjunction with 'at a guaranteed price' and 'brochure accuracy' page 64.

Supplements for fewer adults sharing are shown below and relate to seasons as follows:
LOW = 01 May–23 May, 26 Sep–31 Oct
MID = 24 May–13 Jul, 29 Aug–25 Sep
HIGH = 14 Jul–28 Aug

Child and infant prices see pages 65.

Airport taxes are included.
See pages 60–63 for flight supplements, timings and dates of operation.

FLIGHTS TO DALAMAN ON MONDAY AND THURSDAY

Unijet

(a) Sean and Beth are planning a holiday in Turkey with their 4-year-old daughter. Their first choice is an all-inclusive holiday in early August. How much would this cost them for a week? For a fortnight?

(b) They decide that they could only afford to go for a week at that time of year. But they wonder whether they could afford a fortnight at another time. What is their cheapest option?

Solution

(a) First, identify the relevant sections of the table: the first part of August falls in the row labelled '17 Jul–09 Aug', so look there. The only all-inclusive option comes under the holiday code 232. Hence the relevant adult prices are £525 (for 7 nights) and £745 (for 14 nights). The price for a child is £125 (for 7 or 14 nights).

Therefore the total cost for a week would be

$$(£525 \times 2) + £125 = £1175.$$

The total cost for a fortnight would be

$$(£745 \times 2) + £125 = £1615.$$

(b) The lowest prices for a fortnight appear in the row labelled '01 May–10 May' and in the column headed '1 Bedroom self-catering'. The price for each adult is £225 and for a child is £45.

Unfortunately, there is a catch hidden away in the small print! This price is based on three/four adults sharing, but since there are only two adults they would each have to pay a supplement of £3.95 per night. Then the total cost would be

$$(£225 \times 2) + (2 \times £3.95 \times 14) + £45 = £605.60.$$

The self-catering studio (holiday code 233) may be a better option. Its cost would be

$$(£255 \times 2) + (2 \times £2.25 \times 14) + £45 = £618.00.$$

The cheapest bed and breakfast option (holiday code 234) works out at

$$(£265 \times 2) + £45 = £575.$$

Overall, this is the cheapest option.

Tables often give information in percentages. The table below indicates how the size of households in Great Britain has changed over a period of nearly 40 years.

Number of people in household	1961 (%)	1971 (%)	1981 (%)	1991 (%)	1998 (%)
1	14	18	22	27	28
2	30	32	32	34	35
3	23	19	17	16	16
4	18	17	18	16	14
5	9	8	7	5	5
6 or more	7	6	4	2	2
Number of households surveyed/millions	16.3	18.6	20.2	22.4	23.6
Average household size (number of people)	3.1	2.9	2.7	2.5	2.4

Thus, in 1961, 14% of households consisted of only one person, compared with 28% in 1998. There were over seven million more households in Great Britain in 1998 than in 1961. You can see a steady rise in the percentage of smaller families and a decline in the percentage of larger families.

Other information can be extracted from the table by doing simple calculations. For example, in 1991, the total number of households surveyed was 22.4 million, so the actual number of four-person households surveyed is found by calculating 16% of 22.4 million, which is 3 584 000 households.

Because 22.4 million is a rounded figure, this number of households will not be exact.

Since each column in the table should include all the households surveyed, the total of all the percentages in a column should be 100%; indeed, for 1998,

$$28 + 35 + 16 + 14 + 5 + 2 = 100\%.$$

However, the column total is not always exactly equal to 100%. All the percentages have been rounded to whole numbers, and this can sometimes introduce rounding errors. For instance, the total of the 1961 column is

$$14 + 30 + 23 + 18 + 9 + 7 = 101\%.$$

Rounding errors are usually very small, so the total should always be very close to 100%.

Sometimes the total percentages for both rows and columns are indicated, as in the table below which shows the percentages of families in Great Britain with different numbers of dependent children (at Spring 1998).

Type of family	Number of dependent children			Total (%)
	1 (%)	2 (%)	3 or more (%)	
Couple	17	37	25	79
Lone mother	6	7	6	19
Lone father	1	1*	-	2
Total	24	45	31	100

* This number indicates lone-father families with two or more children.

In tables like this, the row totals *and* the column totals should *always* add up to the same number. For example, in the table above,

$$\text{total in row } 4 = 24 + 45 + 31 = 100\%,$$

and

$$\text{total in column } 4 = 79 + 19 + 2 = 100\%.$$

Here, the row totals and the column totals both add up to 100%, but in other tables rounding errors might mean that the two totals are not exactly 100% (though they should both be the same).

Try some yourself (5.2.1)

Solutions on page 104.

1 The table below indicates the cooling rate of tea in a teapot.

Time/mins	0	5	10	15	20	25	30	35	40	45	50
Temperature/°C	90	65	60	50	36	35	30	26	25	22	20

(a) How long does it take for the tea to cool to 50°C?

(b) By how much does the temperature drop in the first 20 minutes? By how much does it drop in the second 20 minutes?

(c) How would you describe the pattern of cooling?

2 Look at the timetable in Example 4 on page 13.

(a) If you catch the 16:21 train from Doncaster, how long would it take you to get to Wellingborough?

(b) Suppose you want to arrive in Bedford by 7 pm. What time would you need to leave Derby?

3 Look at the table from the holiday brochure in Example 5 on page 14.

(a) Paul and Linda Potter wish to book a week's stay in a self-catering studio in Turkey, leaving on 6 June. How much will their holiday cost them?

(b) How much would they save if they were in one-bedroom self-catering accommodation?

4 Consider the table about household sizes on page 15.

(a) What was the total number of households surveyed in 1971?

(b) How many of the households surveyed in 1998 consisted of three people?

(c) What is the percentage total for the 1981 column?

5 Consider the table about families with dependent children on page 16.

(a) What percentage of dependent children are in single-parent families?

(b) What percentage of children are in families comprising couples with two or more children?

6 Below is a table summarizing the cigarette-smoking habits of a sample of men in various age groups.

Number of cigarettes smoked per day	Percentage of each age group					
	16–24	25–34	35–49	50–59	60 or over	All aged over 16
None	62	52	52	50	60	55
1–20	18	19	20	21	24	22
Over 20	19	29	28	28	16	23
Number of men surveyed	1850	2560	2470	1960	2150	10 990

(a) Write down the percentage of men aged between 25 and 34 who smoke over 20 cigarettes a day.

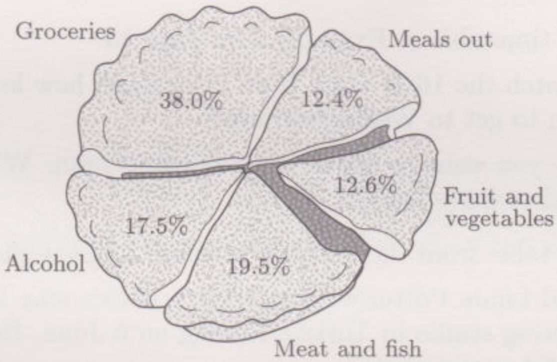
(b) How many of the men aged 60 or over are non-smokers?

(c) Which age group has the highest percentage of heavy smokers?

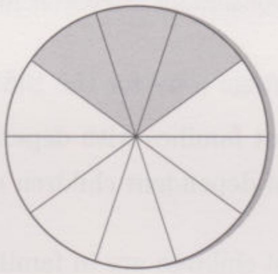
5.2.2 Pie charts

Pie charts are representations that make it easy to compare proportions: in particular, they allow quick identification of very large proportions and very small proportions. They are generally based on large sets of data.

The pie chart below summarizes the average weekly expenditure by a sample of families on food and drink. The whole pie represents 100% of the expenditure. The pie is then divided into 'slices', and the area of each slice represents a fraction or percentage of the total expenditure. For instance, groceries account for 38.0% of the total expenditure. The area of this slice is 38/100 of the total area.



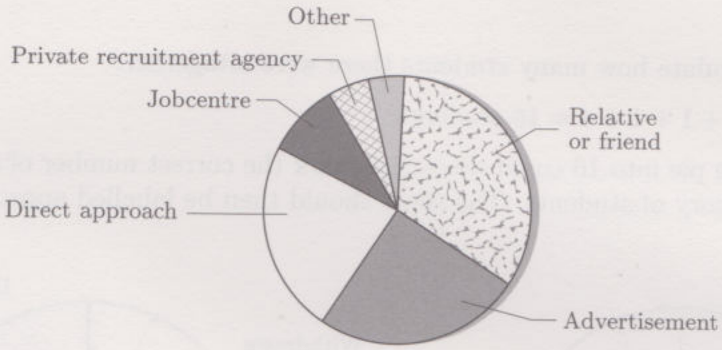
Pie charts can be constructed by dividing a circle into equal slices and then shading in the appropriate fractions. For example, the circle below is divided into 10 equal slices. The shaded area is $\frac{3}{10}$ or 30% of the total.



Often, pie charts do not have the actual percentages or fractions marked on them, so it is difficult to glean any precise information. However, the percentages and fractions *can* be estimated.

Example 6

This pie chart shows how a sample of people first heard about their present jobs. Interpret the pie chart by estimating the percentage of people in each category.



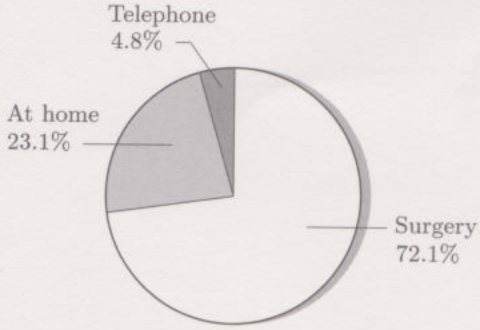
Solution

The percentages can be estimated by eye. Thus the largest proportion (about $\frac{1}{3}$ of the circle or 33%) heard about their jobs from a relative or a friend. About $\frac{1}{4}$ or 25% took the initiative from an advertisement, and a slightly smaller proportion than that, perhaps 22%, followed a direct approach. The three smaller slices are more difficult to estimate: those hearing about their jobs through a Jobcentre correspond to about 10%, while the slices for the private recruitment agency and 'other' are each perhaps half of that, and so individually correspond to about 5% of the total.

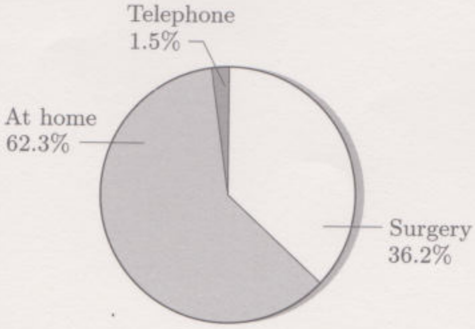
To check these estimates, add the percentages. The total should be close to 100%:

$$33 + 25 + 22 + 10 + 5 + 5 = 100\%.$$

Pie charts can also provide a good visual comparison of proportions arising from different sets of similar data. The pie charts below summarize the methods used by people to consult their doctors. Here the actual percentages are quoted on the slices of the pies. The pie chart on the left presents information relating to people of all ages; the pie chart on the right presents information relating to elderly people. It can be seen that in the population as a whole, most people (72.1%) consult their doctors at the surgery; by contrast, most elderly people (62.3%) are likely to be visited by their doctors at home.



All ages



Over 75

Example 7

In one year a mathematics tutor found that four of her students gained distinctions, six gained Grade 2 passes, one a Grade 3 pass, one a Grade 4 pass and the other four students withdrew during the course. Draw a pie chart to represent these data.

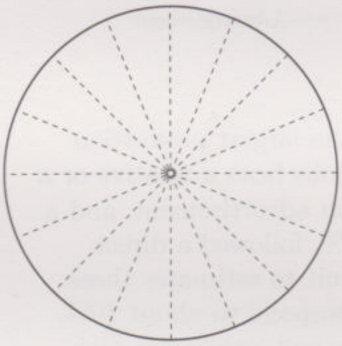
Solution

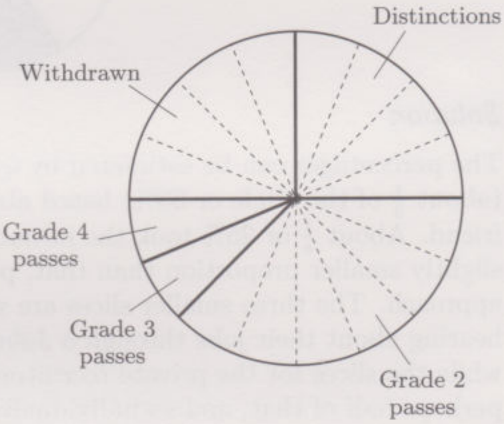
First, calculate how many students there were altogether:

$$4 + 6 + 1 + 1 + 4 = 16 \text{ students.}$$

You will see later how to deal with pie charts where the numbers do not work out so neatly.

Divide the pie into 16 equal slices and mark the correct number of slices for each category of students. The slices should then be labelled appropriately.





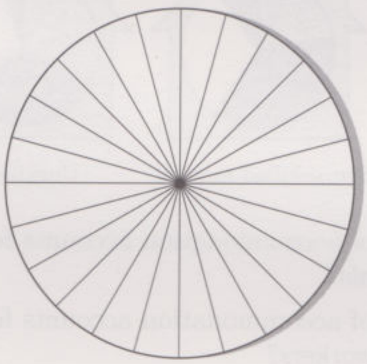
Try some yourself (5.2.2)

Solutions on page 105.

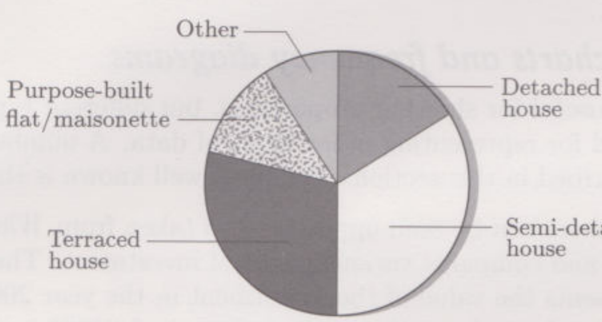
1 This table categorizes Tom's activities for the day.

Activity	Time/hours
Sleeping	8
Working	7
Eating	3
Pub	1
Watching TV	4
Other	1
Total	24

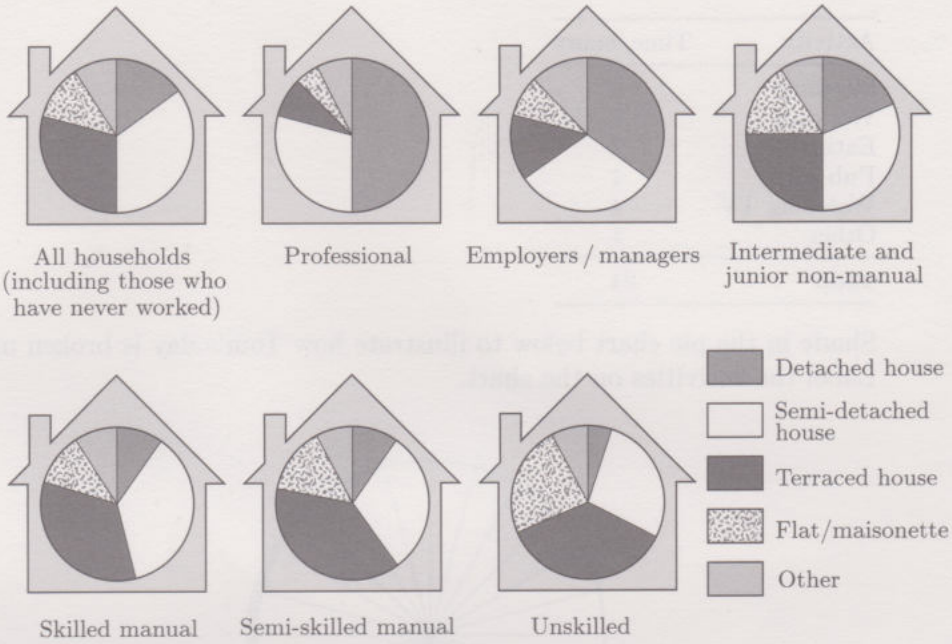
Shade in the pie chart below to illustrate how Tom's day is broken up. Label the activities on the chart.



2 The pie chart below shows the proportions of people living in different kinds of accommodation in a particular town. Interpret what the pie chart indicates by estimating the percentage of people in each category.



3 This diagram represents the proportions of people living in various kinds of accommodation. Separate pie charts have been drawn for people in different types of occupation.



(a) Which type of accommodation accounts for the largest proportion of professionals?

(b) Which type of accommodation accounts for the largest proportion of unskilled workers?

(c) Overall, which is the most common type of accommodation?

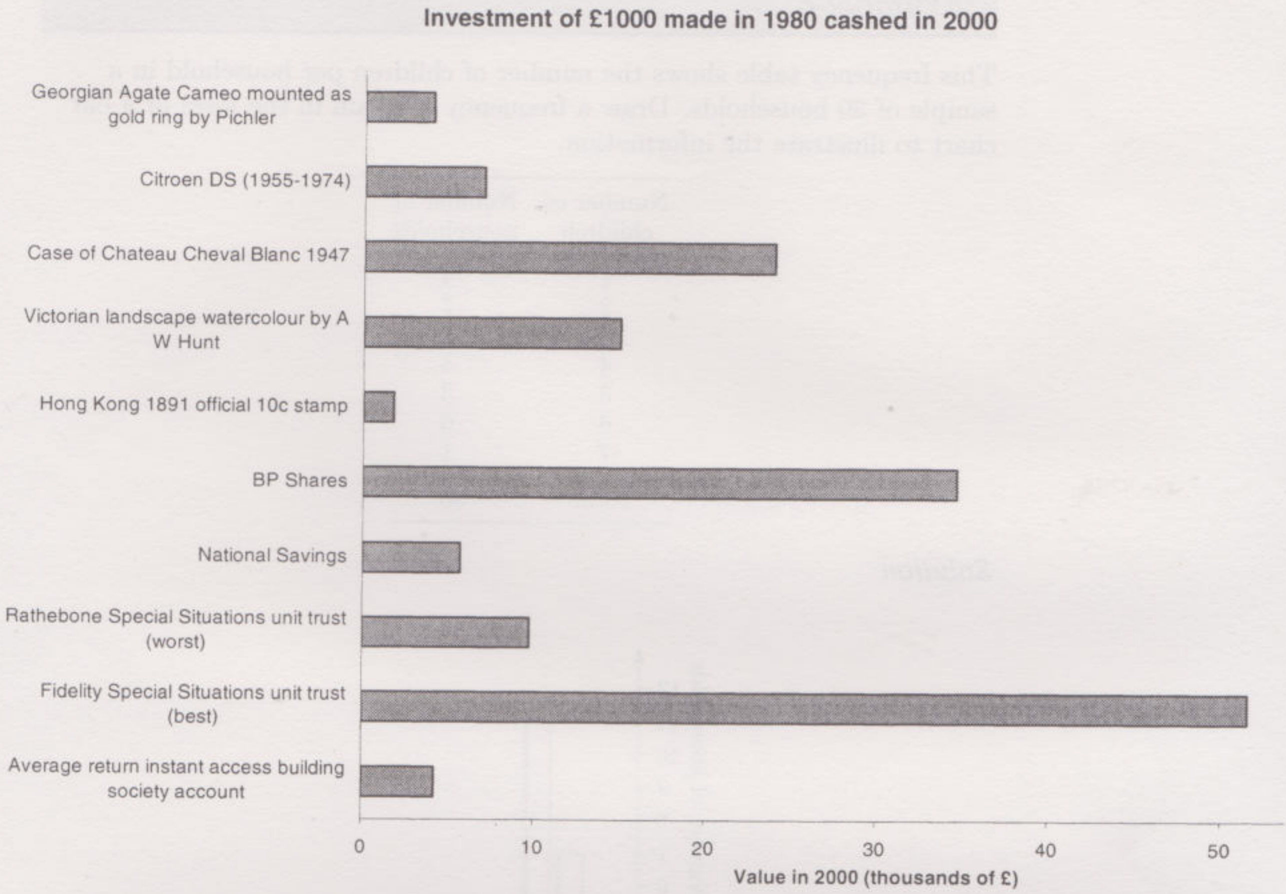
(d) Compare the accommodation used by employers/managers with that used by skilled manual workers.

5.2.3 Bar charts and frequency diagrams

Pie charts are useful for showing proportions, but different types of chart have to be used for representing other kinds of data. A number of these charts are described in this section. The most well known is the **bar chart**.

A typical bar chart can be seen opposite: it is taken from *Which?* (August 2000) and compares various forms of investment. The length of each bar represents the value of the investment in the year 2000. Among other things, the chart shows that an investment of £1000 in a case of Chateau Cheval Blanc (wine) in 1980 was worth £24 000 in 2000, and that the Fidelity Special Situations unit trust was the best investment, as indicated by the longest bar.

The bars on a bar chart are usually drawn not touching one another. Furthermore, to prevent a bar chart giving a misleading representation of the data, the bars should be the same width, and the scale (in this case, the value of the investment in 2000) should start from zero and be clearly labelled.



The bar chart above represents the worth of a range of very different things. Here, a numerical value is associated with each category. In a similar way, a bar chart could be drawn representing, for example, the distances travelled in one hour by different modes of transport: on foot, by bicycle, etc. Again, a numerical value (distance travelled) would be associated with each category (foot, bicycle, etc.). For data of this kind, bar charts are usually the most suitable form of representation.

Another kind of data relates to *how many* items there are in a particular category. For instance, how many days in a year had various types of weather, how many people there are in different age groups in the UK, how many gold medals were won by different countries in the Olympic Games. Such information is often presented in tables that are known as **frequency tables**—the frequencies are the number of times that each possibility occurred.

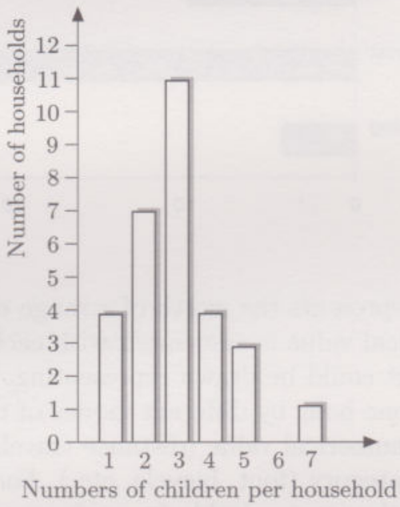
In some cases, the information given in a frequency table can be represented by a bar chart, but in other cases the nature of the data means that another kind of chart (as described later in this section) is required. For convenience, diagrams produced from frequency tables, whether bar charts or some other kind of chart, are all called **frequency diagrams**.

Example 8

This frequency table shows the number of children per household in a sample of 30 households. Draw a frequency diagram in the form of a bar chart to illustrate the information.

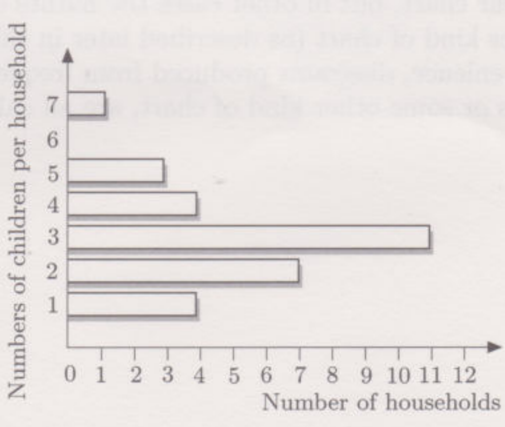
Number of children	Number of households
1	4
2	7
3	11
4	4
5	3
6	0
7	1
Total	30

Solution



The number of children per household has been represented along the horizontal axis, and the number of households up the vertical axis. The height of each bar represents the number of households containing that number of children.

The bars in a bar chart can be drawn either horizontally or vertically. For example, the bar chart in Example 8 could have been drawn as follows:



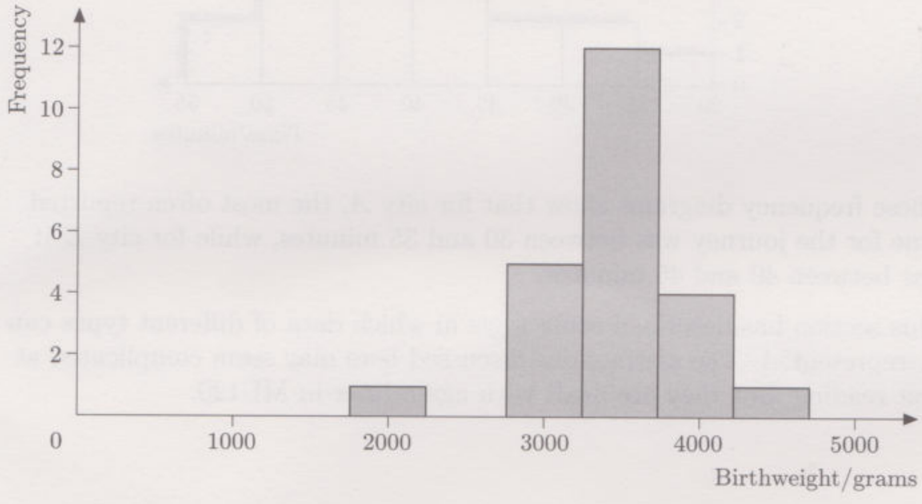
It is important to appreciate that frequency diagrams in the form of bar charts are used only when data are collected in separate categories. Such data, called **discrete data**, are gathered by *counting* things. They include data that can only take certain values and cannot have intermediate values (for example, family size, which can only take whole numbers, or shoe size, which can only take whole and half numbers).

A different kind of frequency diagram is used to represent **continuous data**—data that are *measured* rather than counted (for instance, people's heights and weights, times taken for journeys or phone-calls). The distinction between discrete and continuous data shows up visually in frequency diagrams according to whether or not there are gaps between adjacent bars: thus, a frequency diagram for *discrete* data (that is, a bar chart) should have these gaps (to emphasize that the data are in separate categories), but a frequency diagram for *continuous* data should be drawn with a continuous scale and with adjacent bars touching (to emphasize the continuous nature of the data).

You will meet the term 'histogram' for some kinds of frequency diagram used to represent continuous data. It is related to the frequency diagrams discussed here, but the term is not used in MU120.

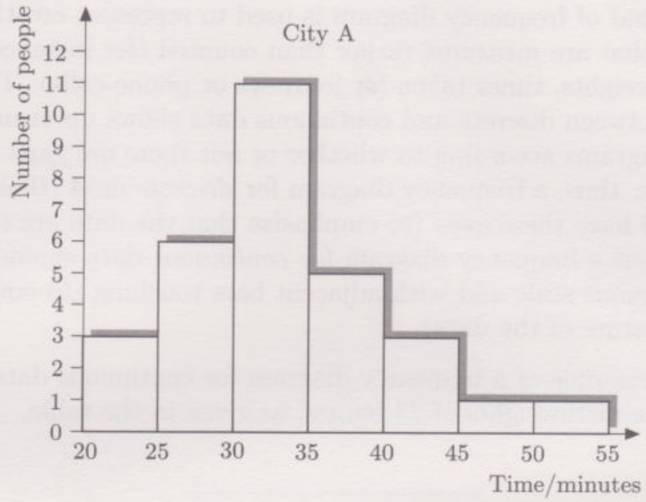
Below is an example of a frequency diagram for continuous data. It represents the birthweights of 23 babies, as given in the table.

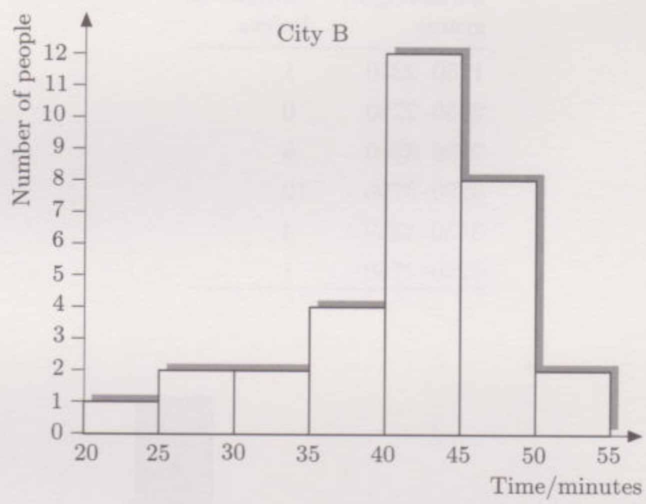
Birthweight/ grams	Number of babies
1750–2250	1
2250–2750	0
2750–3250	5
3250–3750	12
3750–4250	4
4250–4750	1



Notice that frequency diagrams for *continuous* data always have the frequencies shown vertically, while the horizontal scale is continuous. Also notice how the bars are drawn: the lines showing the divisions between each bar are at the boundaries between the groups of data (that is, at 2250 g, 2750 g, 3250 g, etc.). The *height* of each column shows the number of babies whose weights are in each group.

Sometimes you may want to compare the data given in two or more frequency diagrams. The two frequency diagrams for continuous data that are set out below resulted from a survey to investigate the time taken to travel from the centres of two cities, *A* and *B*, to their respective main airports. The journeys of 30 people were surveyed for each city.





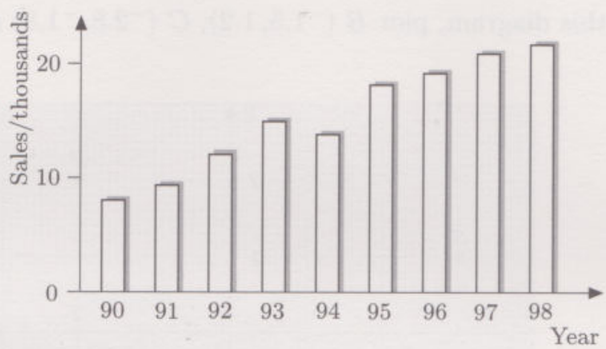
These frequency diagrams show that for city *A*, the most often reported time for the journey was between 30 and 35 minutes, while for city *B* it was between 40 and 45 minutes.

This section has described some ways in which data of different types can be represented. The distinctions discussed here may seem complicated at first reading, but they are dealt with again later in MU120.

Try some yourself (5.2.3)

Solutions on page 105.

1 The bar chart below represents the numbers of cars sold by one company in nine successive years.



(a) How many cars were sold in 1992?

(b) How many cars were sold in 1997?

(c) What does the bar chart suggest about the pattern of sales?

2 This table summarizes the results of a survey of 300 people who were asked how long it took them to get from Central London to Heathrow Airport.

Time/mins	Number of people
30–35	30
35–40	60
40–45	110
45–50	50
50–55	30
55–60	10
60–65	10
Total	300

(a) Illustrate the information by means of a frequency diagram. (You may find it easiest to draw your frequency diagram on graph paper.)

(b) What does the width of each interval represent?

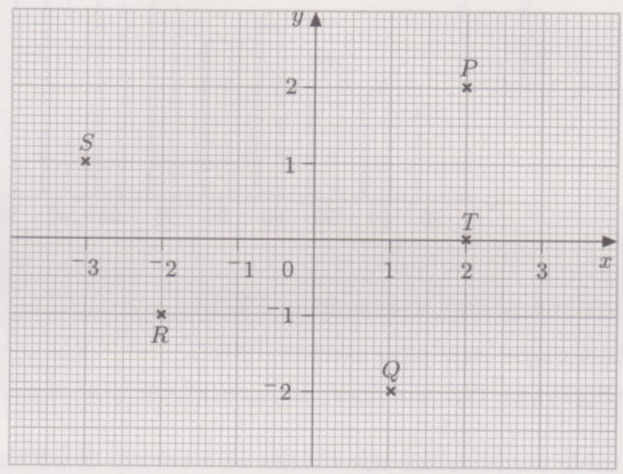
5.3 Coordinates and graphs

These are diagnostic questions. Try them to see which topics you need to revise.

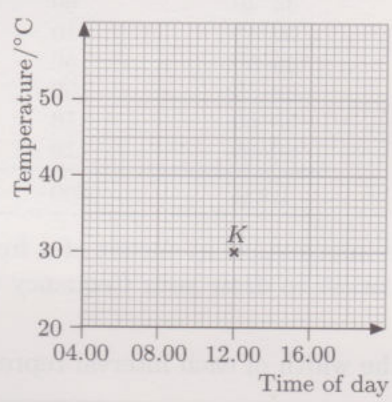
Try these first

1 (a) Write down the coordinates of the points P, Q, R, S and T.

(b) On this diagram, plot B ($^-1.5, 1.2$), C ($^-2.8, ^-1.8$) and D ($0, 2.2$).



2 Write down the coordinates of the point K, and interpret these coordinates in terms of the labels on the axes.



3 Plot the following points on graph paper, choosing and labelling the axes appropriately: $(14, 0)$, $(0, 50)$, $(^-18, 56)$, $(20, 48)$.

4 Draw a graph based on the data in this table. What shape is your graph?

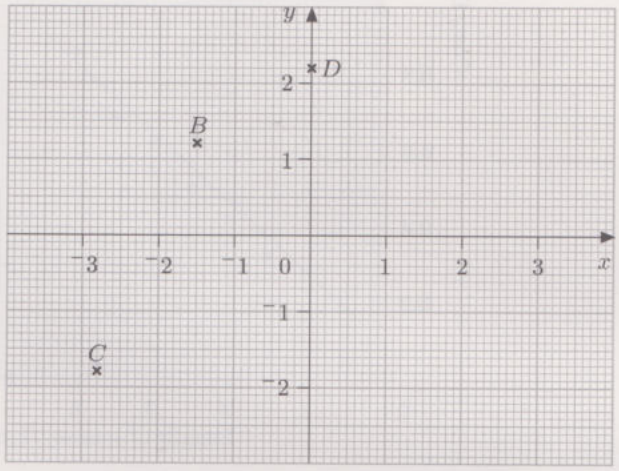
Length of bus journey/km	0.5	1	1.5	2	2.5	3	3.5	4
Cost/£	0.20	0.28	0.32	0.42	0.50	0.62	0.69	0.86

Check your answers

1 (a) $P\ (2,2)$, $Q\ (1,{}^{-}2)$, $R\ ({}^{-}2,{}^{-}1)$, $S\ ({}^{-}3,1)$, $T\ (2,0)$.

Follow-up in Section 5.3.1.

(b)



For clarity, the points P, Q, R, S and T are not shown.

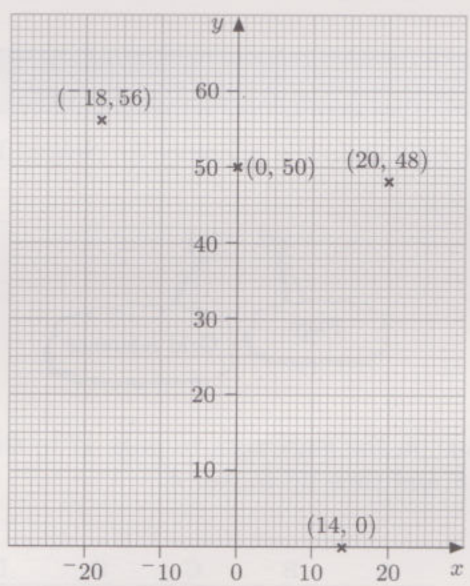
2 The point K has coordinates $(12.00, 30)$. This means that the temperature at midday was 30°C.

Follow-up in Section 5.3.1.

3 First, look at the range of values involved. The horizontal (or x) axis must cover the range ${}^{-}18$ to $+20$, while the vertical (or y) axis must cover the range 0 to 56.

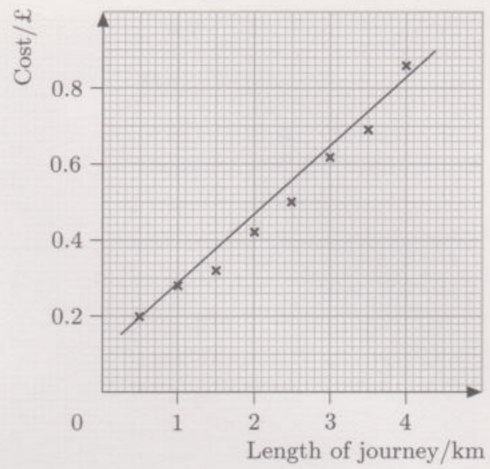
Your answer will depend on the size of the graph paper and the scales you used, but it should look something like this:

Follow-up in Section 5.3.2.



Follow-up in Section 5.3.2.

4 Again, the size of the graph paper and the scales used will have an effect, but the graph should resemble this:



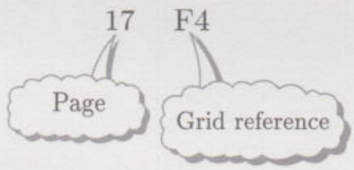
The graph is approximately a straight line.

5.3.1 Coordinates

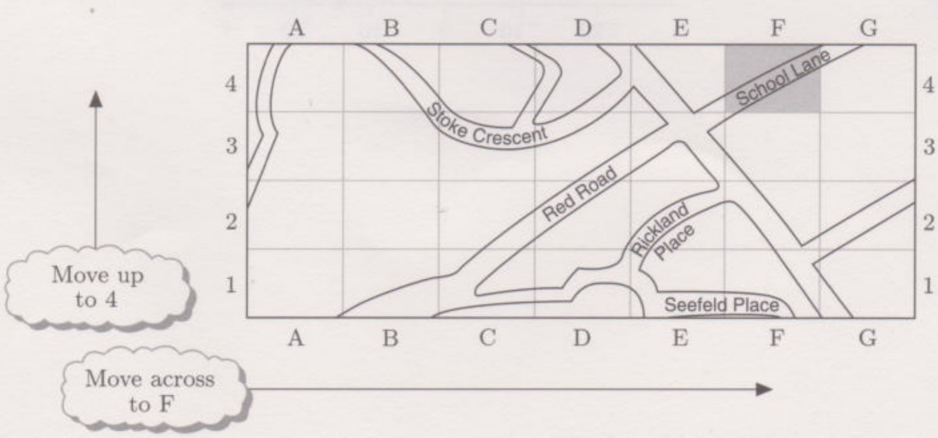
For many towns and cities, an individual book of street maps called an A to Z has been produced. You can look up the name of a street in the index, and it will give you the page number of the map that contains the street, plus the *grid reference* square for the street. There are different conventions for these grid references. You may have met several of these.

Example 9

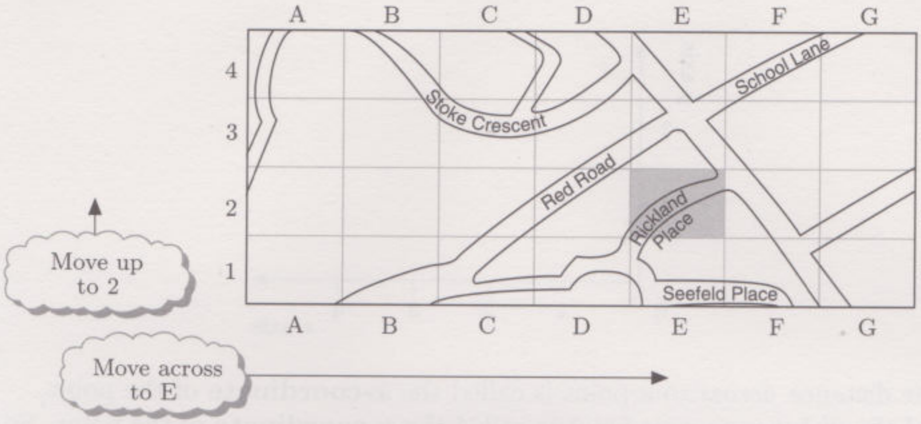
The index in an A to Z gives the reference for School Lane as:



To find School Lane on the map, turn to page 17 in the A to Z, move across to column F and then up to row 4:

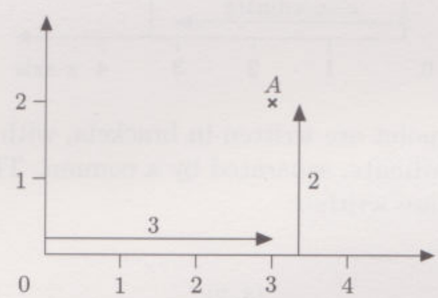


Similarly, the grid reference for Rickland Place on page 17 in the A to Z is E2.

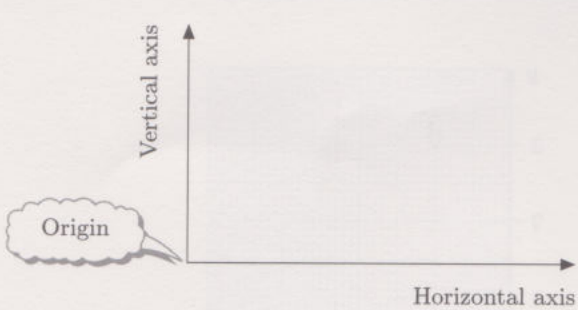


Later in MU120 you will study the convention used for grid references on Ordnance Survey and other maps.

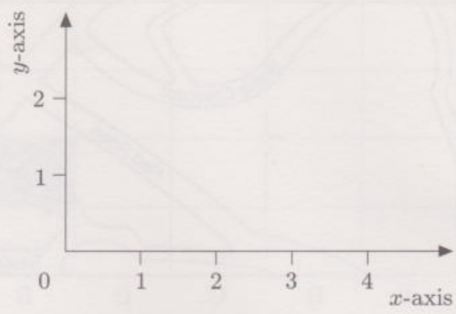
In mathematics, locating points on a grid is similar to finding a place on a map by means of grid references. However, the grid lines themselves are labelled, rather than the squares. For example, on the grid below, the point A is located by moving across 3 and then up 2.



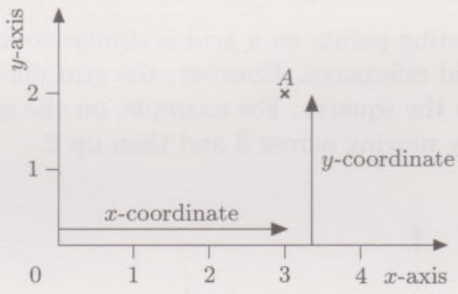
In the same way that the grid references on a map are based on a starting point—in Example 9 the starting point was the bottom left corner of the map (column A, row 1)—a starting point is needed to locate points on a mathematical grid. This starting point is called the **origin**. From the origin you can move horizontally across and vertically up: the line across is called the **horizontal axis**, and the line going up is called the **vertical axis**.



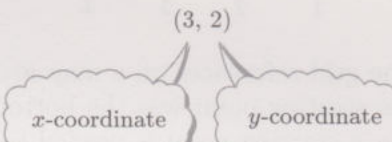
On a mathematical grid, the horizontal axis is often labelled the ***x*-axis**, and the vertical axis is labelled the ***y*-axis**. Scales are indicated on the axes to aid the location of points, and the origin is usually labelled 0.



The distance *across* to a point is called the ***x*-coordinate** of the point, and the distance *up* to a point is called the ***y*-coordinate** of the point. So, in the example below, A is located at the point with x-coordinate 3 and y-coordinate 2.

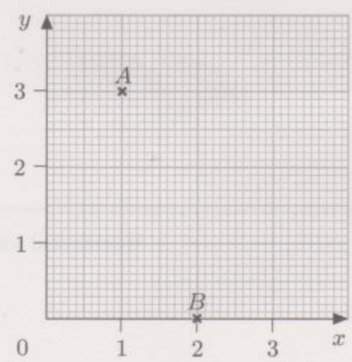


The coordinates of a point are written in brackets, with the x-coordinate followed by the y-coordinate, separated by a comma. Thus the coordinates of the point A above are written:



Example 10

Write down the coordinates of the points A and B.



Solution

To locate A:

start at the origin;
move *across* 1 unit;
move *up* 3 units.

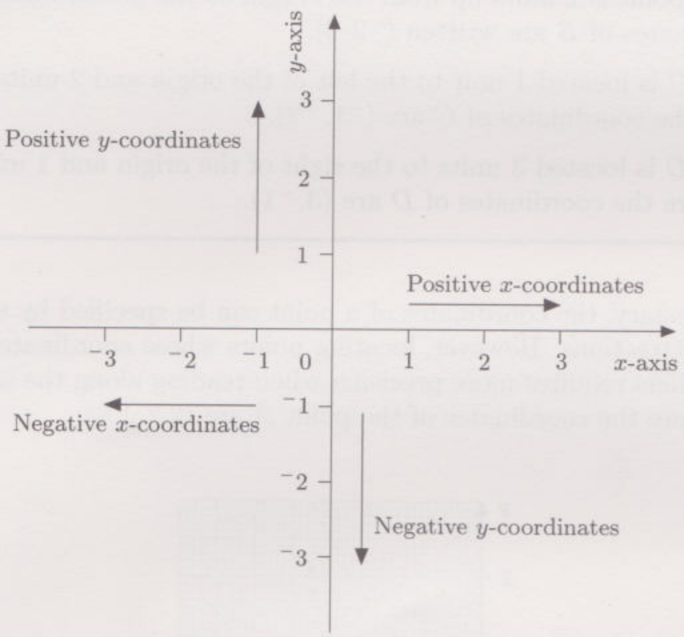
The coordinates of A are $(1, 3)$.

To locate B:

start at the origin;
move *across* 2 units;
move *up* 0 units.

The coordinates of B are $(2, 0)$.

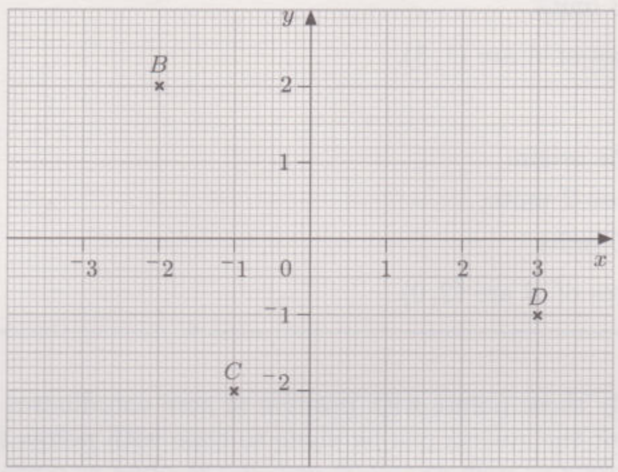
Up to now only those points with positive or zero coordinates have been considered. But the system can be made to cope with points involving negative coordinates, such as $(^-2, 3)$ or $(^-2, ^-3)$. Just as a number line can be extended to deal with negative numbers, the x-axis and y-axis can be extended to deal with negative coordinates.



In this way, if a point is to the left of the origin, its x-coordinate is negative, and if it is below the origin, its y-coordinate is negative. The origin itself is $(0, 0)$.

Example 11

Write down the coordinates of the points B, C and D.



Solution

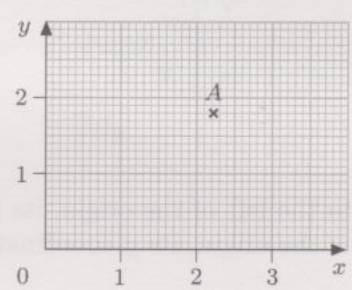
The point B is located 2 units to the left of the origin, so the x-coordinate is $^-2$. The point is 2 units up from the origin, so the y-coordinate is 2. The coordinates of B are written $(^-2, 2)$.

Remember that the x-coordinate is given first, and the y-coordinate is given second.

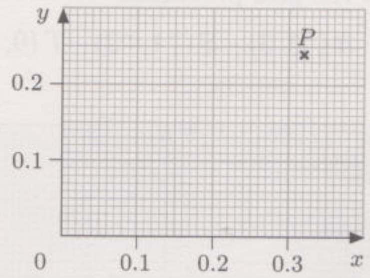
The point C is located 1 unit to the left of the origin and 2 units below it. Therefore the coordinates of C are $(^-1, ^-2)$.

The point D is located 3 units to the right of the origin and 1 unit below it. Therefore the coordinates of D are $(3, ^-1)$.

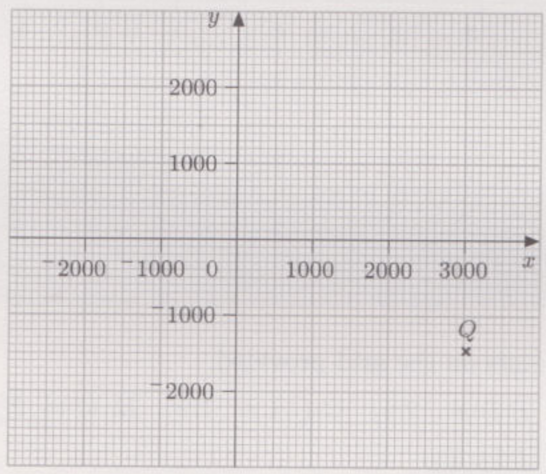
Where necessary, the coordinates of a point can be specified by using decimals or fractions. However, locating points whose coordinates are not whole numbers requires more precision when reading along the axes. For example, here the coordinates of the point A are $(2.2, 1.8)$.



A point P with coordinates $(0.32, 0.24)$ would be particularly difficult to plot accurately on the axes above because of the scales used. You can only plot the point approximately. But, if a larger scale were used for the axes, it would be quite easy to plot P with reasonable accuracy, as shown at the top of the opposite page.



Similarly, to plot points with very large coordinates, such as a point Q with coordinates $(3020, {}^-1450)$, a very different scale would be needed.



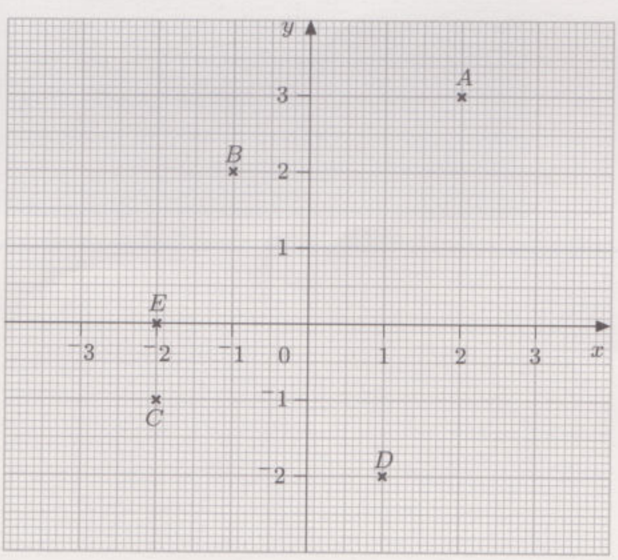
Notice that, even using scales like these, it is impossible to plot points such as Q exactly.

Mathematical coordinate grids are used for maps, plans, geometric diagrams and also for graphs. For the first three of these, the scales on the axes of the grid must be the same (otherwise shapes would be distorted and diagonal distances would be incorrect). For graphs, the scales on the axes need not be the same—in fact, often the two scales represent very different things.

Try some yourself (5.3.1)

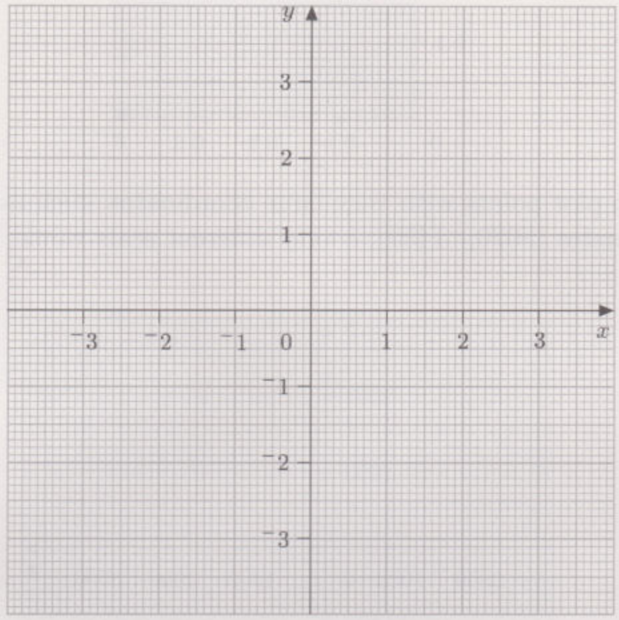
Solutions on page 106.

1 Write down the coordinates of the points A, B, C, D and E.



2 Plot these points on the grid provided:

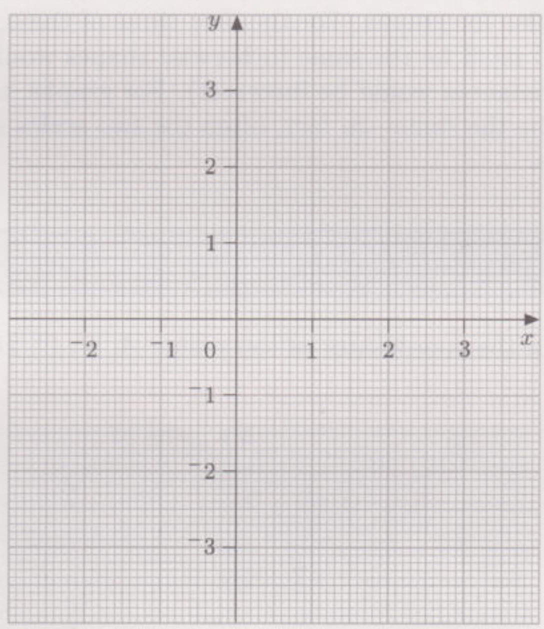
$P\,(2,3)$ $Q\,(^-2,1)$ $R\,(^-3,3)$ $S\,(^-1,^-2)$ $T\,(0,^-3)$ $U\,(3,^-1)$.



3 Plot these points on the grid provided:

$A\,(2\tfrac{1}{2},1)$ $B\,(0.8,^-2.2)$ $C\,(^-1.8,^-1.2)$ $D\,(^-1\tfrac{1}{2},2\tfrac{1}{4})$

$E\,(1.3,2.4)$ $F\,(\tfrac{1}{3},\tfrac{2}{3})$.

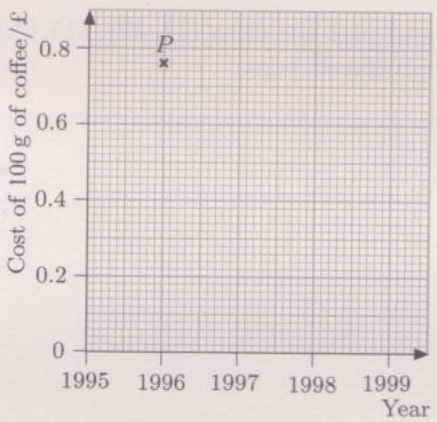


5.3.2 *Drawing and interpreting graphs*

A graph shows the relationship between two quantities. These quantities may be very different: for instance, the price of coffee in relation to different years, or the braking distance of a car in relation to different speeds, or the height of a child at different ages. Because the quantities are different, there is no need to have equal scales on the graph, and it is often impractical to do so. However, it is essential that the scales are shown on the axes: they should indicate exactly what is being measured and the units of measurement used.

Example 12

Write down the coordinates of the point P, and interpret those coordinates in terms of the labelling on the axes.



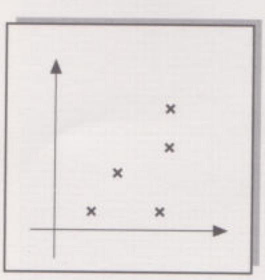
Notice that the horizontal axis starts at 1995, rather than 0.

Solution

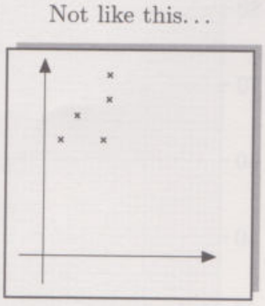
Here the horizontal axis represents years and the vertical axis represents the cost of 100 g of coffee in pounds (£). The coordinates of P are $(1996, 0.76)$, so in 1996 the cost of 100 g of coffee was £0.76 (or 76 p).

If you gather data yourself (perhaps by conducting an experiment or carrying out a survey) and want to represent that data graphically, you will probably have to decide what the axes should denote and what scales to use. This is often the hardest part of plotting a graph, and it is easy to go wrong at first. Ideally, the choices should be such that the points can be plotted and read off easily, and they fill the available space sensibly.

Like this...



Not like this...



Because graph paper is usually marked off in squares of 5 units or 10 units, it makes sense to use scales such as

1 large square: 1 unit

1 large square: 5 units

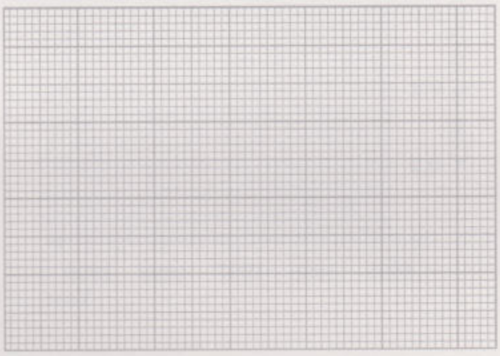
1 large square: 10 units.

Sometimes other choices of scale may be sensible. For example, if you are plotting times, you might choose 1 square to represent 6 seconds, and 10 squares to represent 60 seconds or 1 minute.

It is not always easy to choose scales or to decide where the axes should cross. You will find as you work through MU120 that the graphics calculator makes it very simple to experiment with different scales for the axes.

Example 13

Choose suitable axes and plot the data $(11, 42)$, $(15, 68)$, $(3, 59)$ and $(5, 72)$ on the graph paper reproduced here.

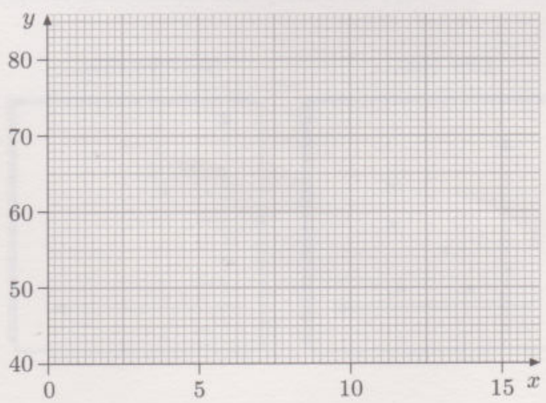


Solution

First, look at the range of the data. The x-coordinates range from 3 to 15: the smallest is 3, and the largest is 15. This suggests that the x-axis should range from 0 to 15. When the range of coordinates is compared with the size of the graph paper, a suitable scale for the x-axis seems to be 2 large squares to 5 units.

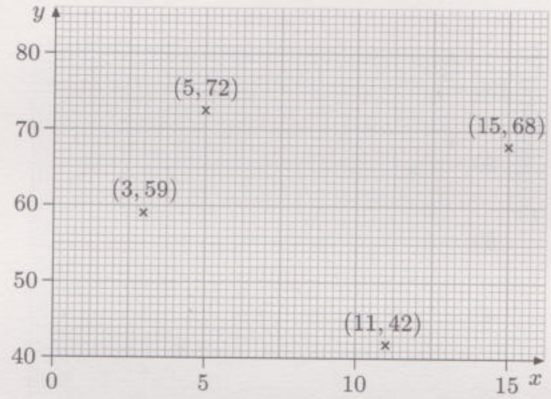
The y-coordinates range from 42 to 72. You could start at 0 and use a scale of 5 small squares to 10 units. But it is probably more practical to start the scale at 40 since all the y-coordinates are greater than 40, and use a scale of 1 large square to 10 units.

The origin, $(0, 0)$, does not necessarily have to be included. As long as the axes are clearly labelled, there will be no confusion.



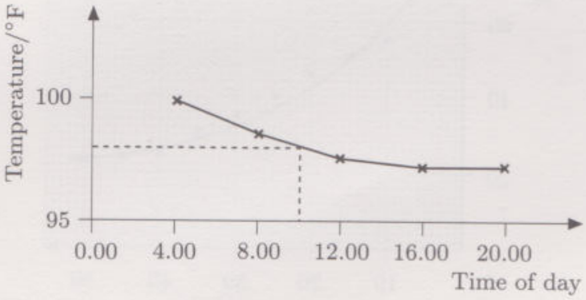
Now plot the first of the points, (11, 42). On the horizontal axis, 20 small squares represent 5 units, so 4 small squares represent 1 unit. Therefore, 11 is 4 small squares to the right of the point labelled '10'. On the vertical axis, 10 small squares represent 10 units, so 42 is 2 small squares above the point labelled '40'.

The other points can be plotted in a similar way:



A set of plotted points can be joined up by a line or a curve. The resulting graph provides more information than the isolated points. It gives a better picture of a relationship and sometimes allows you to predict values in between the given points.

For example, the temperature chart below indicates the hourly progress of a patient. Experience suggests that there should not be any major fluctuations between the points marked, so it is reasonable to join up these points. You can then see clearly how the patient's temperature dropped and approached a normal value. Although the temperature was not taken at, say, 10.00, the graph indicates that it was about 98°F at that time.

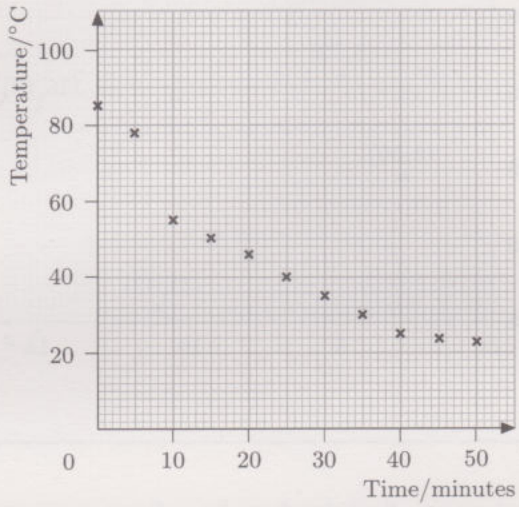


In this case, the points have been joined up using straight lines. In other cases, points may be joined to give a smooth curve.

It is often easier to visualize relationships by using a graph rather than a table of data, as the following example shows. The data in this table resulted from an experiment to investigate how quickly a cup of tea cooled.

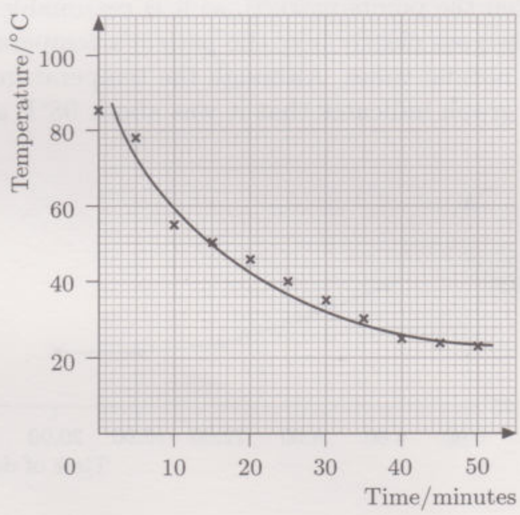
Time/mins	0	5	10	15	20	25	30	35	40	45	50
Temperature/°C	85	78	55	50	46	40	35	30	25	24	23

When plotted, the data produced these points:



The points almost lie on a smooth curve—but not exactly. In such cases the graph is completed by drawing the smoothest curve possible. The graph illustrating the cooling rate of tea can therefore be completed as follows:

In MU120 you will learn how a graphics calculator can be used to produce a curve that fits the data well.



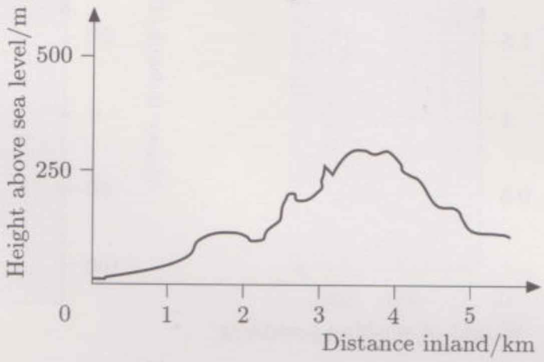
You can see at a glance that the temperature drops quickly at first, and subsequently its rate of decrease slows down.

Temperature and time are the *variables* in the graph above. Temperature is measured up the vertical axis, and time is measured along the horizontal axis. It is not always obvious which variable should be measured along which axis (though it is usual to measure time along the horizontal axis). It often does not matter: the important thing is that the axes are labelled and the units of measurement are clearly indicated, making it possible to interpret the graph correctly.

Example 14

In this graph, ground height above sea level is plotted against distance from the coast measured along a straight line running inland in an east–west direction. Interpret the graph.

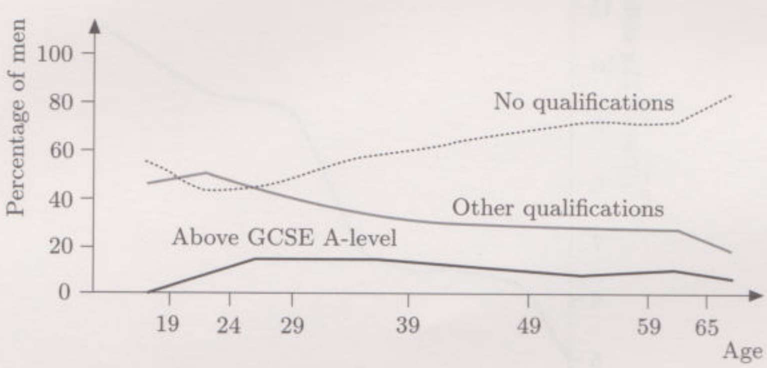
You will see how to produce graphs like this in *Unit 6* of MU120.



Solution

The graph shows that, starting at sea level, the height rises gently at first, so initially the terrain is quite flat; then, after about 1 km, it rises irregularly, eventually reaching about 300 m after approximately 3.5 km, before falling again.

Sometimes several graphs are plotted together on the same axes so that the reader can *compare* the information. For example, the figure below shows graphs of the percentages of men in a certain community with various qualifications relative to age.



These graphs can be analysed separately. But it is also possible to compare the information from them, as the following observations indicate:

- At the age of 49,

  about 10% of the men have qualifications above A-level,
  about 30% have some other qualification,
  about 65% have no qualifications.

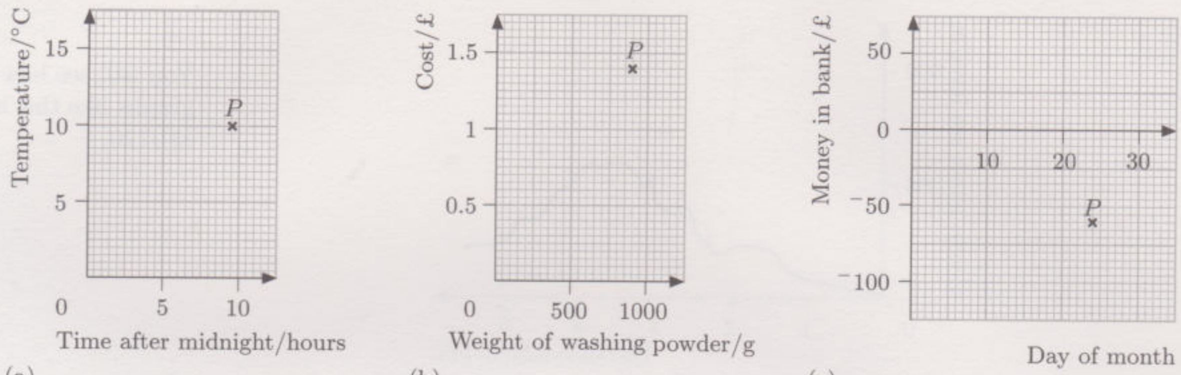
- A higher percentage of younger men (aged over 20) have some form of qualification compared with older men. (This perhaps reflects the availability of a greater number and variety of educational opportunities compared with earlier times.)
- The percentage of men with qualifications above A-level remains fairly constant for men aged over 26.

Notice that the total percentage is about $10 + 30 + 65 = 105\%$. The discrepancy arises because percentages can only be read approximately from the graph.

Solutions on page 106.

Try some yourself (5.3.2)

1 Write down the coordinates of each of the points P and interpret those coordinates in terms of the labels on the axes.



(a) (b) (c)

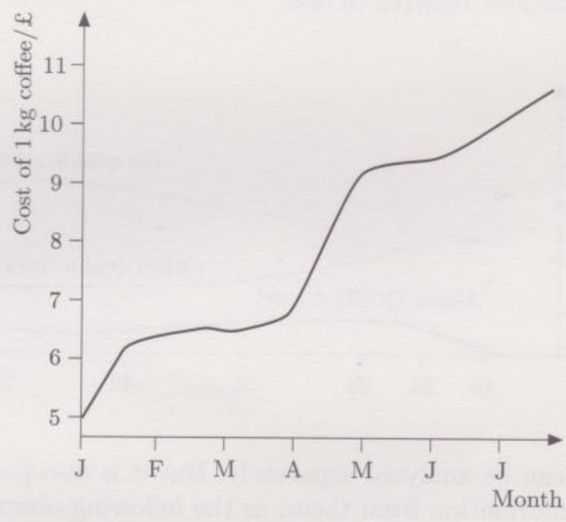
2 On graph paper, plot each of the following sets of points on suitable axes:

(a) $(2, 10)$, $(4, 38)$, $(7, 26)$, $(5, 23)$;

(b) $(350, 150)$, $(420, 168)$, $(630, 172)$, $(570, 159)$;

(c) $(140, 6)$, $(^{-}100, 30)$, $(60, 13)$, $(^{-}60, 22)$.

3 Summarize the information given by the graph below.



4 This table records the value of a particular car as it gets older. Plot a graph to illustrate the relationship, drawing a curve through the points plotted.

Year	1992	1993	1994	1995	1996	1997	1998	1999
Price/£	10 000	7000	5000	3500	2300	1800	1300	1000

Outcomes

Now that you have studied Module 5 you should be able to:

- ◇ draw and interpret scale diagrams;
- ◇ extract information from tables;
- ◇ draw, interpret and compare pie charts, bar charts and frequency diagrams;
- ◇ use and interpret coordinates;
- ◇ plot points and draw graphs, using suitable axes and scales;
- ◇ interpret and compare graphs.

Module 6 Language, notation and formulas

The first section of this module deals with some of the rules for writing and reading mathematics. It looks at the conventions for laying out a mathematical argument and considers some of the symbols used.

The second section explores the important idea of formulas—in this case, concentrating on word formulas rather than the algebraic forms that are dealt with in the main MU120 course.

6.1 Communicating mathematics

Try these first

These are diagnostic questions. Try them to see which topics you need to revise.

1 Read the following expressions out aloud or write them out in full in words:

(a) $21 + 34 = 55$ (b) $3 \times 4 + 3 \times 5 = 3(4 + 5)$.

2 Here is a poor example of mathematical writing, although the final answer is correct. Rewrite it, correcting the layout and the mathematical punctuation, and add some words of explanation.

$$\frac{3^2 + 4^2}{5^2} \quad = 5^2 = 25 \quad = 4^2 = 16 \quad = 3^2 = 9 \quad = \frac{9 + 16}{25} = \frac{25}{25} = 1$$

3 Which mathematical operations are implied by these words when used in a mathematical sense?

(a) Sum

(b) Difference

(c) Product

(d) Quotient

State the operations in both words and symbols.

4 What do these symbols mean?

(a) $=$

(b) $<$

(c) $>$

(d) $\simeq$

Check your answers

Follow-up in Section 6.1.1.

1 (a) Twenty-one plus thirty-four equals fifty-five.

(b) Three times four, plus three times five equals three times the sum of four and five.

2 There are many possible alternatives—here is one of them.

Follow-up in Section 6.1.1.

Since

$$5^2 = 25, \quad 4^2 = 16 \quad \text{and} \quad 3^2 = 9,$$

it follows that

$$\frac{3^2 + 4^2}{5^2} = \frac{9 + 16}{25} = \frac{25}{25} = 1.$$

3 (a) Sum implies addition or $+$.

Follow-up in Section 6.1.2.

(b) Difference implies subtraction or $-$.

(c) Product implies multiplication or $\times$.

(d) Quotient implies division or $\div$.

4 (a) $=$ means 'equals' *or* 'equal to' *or* 'which equals'.

Follow-up in Section 6.1.3.

(b) $<$ means 'less than' *or* 'which is less than'.

(c) $>$ means 'greater than' *or* 'which is greater than'.

(d) $\simeq$ means 'is approximately equal to'.

6.1.1 Layout

Writing mathematics has a lot in common with writing English. When you write mathematics, you should write in the equivalent of sentences, with full stops at the end. As in English, each new statement should follow on logically from the previous one or it should contain an indication that a new idea is being introduced. However, *laying out* mathematics differs from laying out English: because mathematics is more condensed than written English, it is often best to start each statement on a new line so that the various steps involved are readily apparent.

In writing mathematics you can use a mixture of words and symbols—symbols in mathematics are just shorthand for words. If you do not know the correct symbol, use the word itself.

Above all, make sure that you write so that somebody else can understand—that somebody might be yourself at a later date! It often helps to read aloud the mathematics that you have written to see if it makes sense.

Example 1

Read the following mathematics aloud:

(a) $24.67 - 12.45 = 12.22$

(b) $3 \times 4 = 4 \times 3 = 12$.

Solution

(a) Twenty-four point six seven minus twelve point four five equals twelve point two two.

(b) Three times four equals four times three which is equal to twelve.

One of the most misused mathematical symbols is the equals sign, $=$. It stands for the verb 'equals' or the phrase 'is equal to' or 'which equals', and so it should only come between two things that are equal.

Example 2

Which of the equals signs should not be included in these calculations?

(a) $24.67 - 12.45 = 12.22$

(b) $6.4 + 23.34 = 29.74$

(c) $= \dfrac{24.67 - 12.45}{6.4 + 23.34} = \dfrac{12.22}{29.74}$

Solution

The equals signs in (a) and (b) are used correctly, but the first sign in (c) is not. This equals sign is not between two things that are equal, and consequently it should not be there. Also it is at the beginning of a sentence, and when you read it aloud it does not make sense.

A lot of people use the equals sign wrongly in places where another word or phrase might actually make the meaning clearer. Sometimes a link word or phrase is useful at the beginning of a mathematical sentence: examples include 'So', 'This implies' or 'It follows that'. One of these might have been suitable instead of the inappropriate equals sign in the above example.

Example 3

Add link words and punctuation to the calculation below to help somebody else to follow it.

$24.67 - 12.45 = 12.22$

$6.4 + 23.34 = 29.74$

$\dfrac{24.67 - 12.45}{6.4 + 23.34} = \dfrac{12.22}{29.74} = 0.410\,89$ (to 5 d.p.)

Solution

Since

$$24.67 - 12.45 = 12.22$$

and

$$6.4 + 23.34 = 29.74,$$

it follows that

$$\begin{aligned}\frac{24.67 - 12.45}{6.4 + 23.34} &= \frac{12.22}{29.74}\\ &= 0.410\,89 \text{ (to 5 d.p.).}\end{aligned}$$

Notice that mathematical calculations are often easier to read if all the equals signs are aligned underneath one another.

Sometimes you may want to refer to mathematical sentences further up your work. You can label such sentences and then refer back by label. Thus, Example 3 could be laid out as follows.

$$24.67 - 12.45 = 12.22, \tag{1}$$
$$6.4 + 23.34 = 29.74. \tag{2}$$

So, from (1) and (2),

$$\frac{24.67 - 12.45}{6.4 + 23.34} = \frac{12.22}{29.74}$$
$$= 0.410\,89 \text{ (to 5 d.p.).}$$

Try some yourself (6.1.1)

Solutions on page 107.

1 In the following two pieces of mathematical writing, remove or replace any inappropriate equals signs, and add link words and punctuation to help somebody else understand the mathematics.

(a) $2.3 + 3.7 = 6 \quad = 14.8 - 5.6 = 9.2$
$$= \frac{2.3 + 3.7}{14.8 - 5.6} = \frac{6}{9.2} = 0.65 \text{ (to 2 d.p.)}$$

(b) $(3.2)^2 = 10.24 \quad = (8.5)^2 = 72.25 \quad = 10.24 + 72.25 = 82.49$

2 Two labels have been omitted in the mathematics below. Where should they go to make sense of the argument?

Since

$$2.3 + 3.7 = 6 \quad \text{and} \quad 14.8 - 5.6 = 9.2,$$

it follows that

$$\frac{2.3 + 3.7}{14.8 - 5.6} = \frac{6}{9.2} = 0.652\,173\,913.$$

Now

$$(3.2)^2 = 10.24 \quad \text{and} \quad (8.5)^2 = 72.25.$$

Hence

$$(3.2)^2 + (8.5)^2 = 10.24 + 72.25 = 82.49.$$

So, from (1) and (2),

$$\frac{2.3 + 3.7}{14.8 - 5.6} + (3.2)^2 + (8.5)^2 = 0.652\,173\,913 + 82.49$$
$$= 83.14 \text{ (to 2 d.p.).}$$

6.1.2 Vocabulary

In mathematics, some words are used in a more precise way than in English. It is important that a mathematical argument is unambiguous; therefore words that can be used in several contexts in English usually take only one meaning in mathematics. For instance, in English the word 'sum' might mean any calculation, but it has a precise mathematical meaning as exemplified by 'The sum of 456 and 789 is 1245'. Similarly, in English the word 'product' can have a variety of meanings, but in mathematics the product of two numbers is the result obtained by multiplying those numbers together.

Overleaf is a list of mathematical words and phrases, together with their mathematical meanings and examples of their use. You will have come across some of them before; others may be new to you. Read them carefully and try to see the connections with the meanings that the words have in everyday English. You may well be able to add some other mathematical terms to the list now or in your future studies.

Many of the geometric terms are discussed in Module 7.

Word/phrase	Mathematical meaning	Example of use
Even number	A whole number that is exactly divisible by two (that is, it results in a whole number when divided by two).	432 is an even number.
Odd number	A whole number that is not exactly divisible by two.	321 is an odd number.
Sum	The addition of numbers.	The sum of 123 and 456 is 579.
Difference	The numerical difference between two numbers, *or* the positive result of subtracting one number from the other.	The difference between 24 and 42 is 18. The difference between 5 and $^{-}2$ is 7.
Product	The result of multiplying numbers together.	The product of 3, 4 and 5 is 60.
Quotient	One number divided by another.	The quotient $60 \div 12$ is 5.
Exponent	The power to which a number is raised.	In the expression $(2.6)^4$, 4 is the exponent.
Perimeter	The outer boundary of a geometric figure (also the length of that boundary).	The perimeter of the figure was drawn in red.
Circle	A geometric figure in which every point on its perimeter is the same distance (the radius) from its centre.	The circle has a radius of 4 cm.
Circumference	The perimeter of a circle or other curved geometric figure (also the length of that perimeter).	The circumference of this circle is 34 mm.
Arc	Part of the circumference of a circle or other curved geometric figure.	The length of the arc of a quarter circle is a quarter of the circle's circumference.
Right angle	The angle between two perpendicular lines; 90°.	The angles at the corners of a square are right angles.
Parallelogram	A geometric figure that has four straight sides; its opposite sides are of equal length and are parallel.	A rectangle is a parallelogram with right angles at its corners.
Similar	The same shape.	All squares are similar, but not all rectangles are similar.
Congruent	The same shape and the same size.	Congruent triangles have the same shape and size.
Equation	One expression equalling another.	The equation $6 - 9 = 9 - 3$ is incorrect.
Inequality	One expression being less than or greater than another.	The inequality $3 < 6$ is correct.

Try some yourself (6.1.2)

Solutions on page 107.

1 Add the following words to the list opposite:

decimal, fraction, positive, negative.

For each one, give the mathematical meaning and an example of its use.

6.1.3 Making sense of symbols

Mathematical symbols are a shorthand way of writing words or phrases that crop up often in mathematical writing. This section looks at arithmetical symbols, some of which are in common usage and others of which may be unfamiliar to you.

You will have already met the symbols for the **basic arithmetical operations**, which are $+$, $-$, $\times$ and $\div$, but you may not have met some of the alternative ways of writing $\times$ and $\div$.

To recap, the main symbols for arithmetical operations are:

$+$ means 'plus' *or* 'add' *or* 'and';
$-$ means 'minus' *or* 'subtract' *or* 'take away';
$\left.\begin{matrix}\times\\ *\end{matrix}\right\}$ both symbols mean 'times' *or* 'multiplied by';
$\left.\begin{matrix}\div\\ /\end{matrix}\right\}$ both symbols mean 'over' *or* 'divided by'.

Remember not to confuse the subtract sign ($-$) with the negative sign ($^-$) used for negative numbers.

There are other alternatives to $\times$ and $*$ for multiplication. Sometimes, provided there is no risk of ambiguity, no symbol is used, just as sometimes the words 'multiplied by' or 'times' can be omitted in speech: for example, two fours or five fives means two times four or five times five, respectively. So, instead of writing $3 \times (4+5)$, you can just write $3(4+5)$ and save one symbol. Mathematicians like to be as concise as possible!

An alternative to $\div$ and / for division is a fractional notation: for example, $\frac{3+6}{4}$ means 'add three and six, and then divide by four'. The slash symbol / is sometimes used for fractions: thus, three-quarters may be written as 3/4. Many calculators use this notation to display fractions.

Another useful form of notation, which you have already come across, are brackets; these serve to avoid ambiguity in mathematical expressions.

The use of brackets was discussed in Module 1.

In Module 4, you encountered several of the **symbols for powers and roots**: for instance, 2^4 means '2 to the power 4'. An alternative to 2^4 is 2^4, where the symbol ^ means 'to the power'. This symbol is often used by computers and is also employed by the course calculator.

A related symbol is the square root symbol, as in $\sqrt{25}$ meaning the square root of 25 (that is, 5). Later in the course you will meet the symbols for other roots, as exemplified by $\sqrt[3]{8}$ meaning the cube root of 8 (that is, 2).

In Module 4, you came across the symbol for a reciprocal: thus, 6^{-1} means the reciprocal of 6, or $\frac{1}{6}$. You also encountered the symbols used by the calculator for scientific notation: for example, 3.1E2 means 3.1×10^2, or 310.

Example 4

What do the following expressions mean?

(a) $5/(3+2)$ (b) $5/3+2$ (c) $(5+8)^2$ (d) $4(8-5)$ (e) $\sqrt{(10+6)}$

Solution

(a) Divide 5 by the sum of 3 and 2. (This gives 1.)

(b) Divide 5 by 3 and then add to 2, *or* $\frac{5}{3}$ added to 2. (This gives $3\frac{2}{3}$.)

(c) Add 5 and 8, and then square the result. (This gives 169.)

(d) Multiply 4 by the difference between 8 and 5. (This gives 12.)

(e) The square root of the sum of 10 and 6. (That is, 4.)

There are also **symbols that show the relationship between numbers or quantities**. Two common ones that you have met before are $=$ and $\simeq$, but there are several other symbols of this type which are used in MU120.

Symbols indicating relationships include:

$=$ means 'equals';
$\neq$ means 'does not equal';
$\simeq$ means 'is approximately equal to';
$<$ means 'is less than';
$>$ means 'is greater than';
$\leq$ means 'is less than or equal to';
$\geq$ means 'is greater than or equal to'.

Another type of symbol that you have already come across are the abbreviations for units of measurement: for example, m for metres, kg for kilograms, and h for hours.

Example 5

What do the following mean?

(a) time > 2 s (b) height ≤ 100 m (c) $\frac{2}{3} \simeq 0.67$

Solution

(a) The time is greater than 2 seconds.

(b) The height is less than or equal to 100 metres.

(c) Two-thirds is approximately equal to 0.67.

Other symbols that are sometimes used (though not in MU120) include:

$\doteqdot$ means 'is approximately equal to' (an older version);
$\equiv$ means 'is exactly equivalent to';
$\Rightarrow$ means 'implies';
$\therefore$ means 'therefore';
$\because$ means 'because'.

You will meet many symbols in your mathematical studies. It is important that you are sure what each one means in any given context, so that you can read it and use it appropriately.

Try some yourself (6.1.3)

Solutions on page 107.

1 What do the following mean?

(a) $(5+8)/(4-2)$ (b) $5+8/4-2$ (c) $(4+5)(5-2)$

(d) $9\sqrt{4}$ (e) $\dfrac{5*6}{2}$ (f) $(\sqrt{25})$^3 (g) $(12-9)^{-1}$

2 What do the following mean?

(a) mass $\geq 10\,\text{kg}$ (b) time $< 2.4\times10^6\,\text{h}$ (c) $2/3 \neq 0.67$

6.2 Formulas

Try these first

These are diagnostic questions. Try them to see which topics you need to revise.

1 Suppose you are selling a new lawn-edging strip and need to calculate the perimeters of lawns in order to quote prices to prospective customers.

The formula for the distance round the perimeter of a rectangular lawn is

$$\text{perimeter} = (2\times\text{length}) + (2\times\text{breadth}).$$

Use this formula to find the perimeter of a lawn that is 10 m by 8 m.

2 You sell this lawn-edging strip by the metre, but some customers have measured their lawns in feet so you have to convert their measurements into metres.

You know that the perimeter formula works whatever unit is used for the length and the breadth. But you decide to check whether it makes a difference if you convert to metres *before* or *after* you apply the formula.

(a) Use 1 foot = 0.3048 m to convert the measurements of a lawn that is 12.5 feet by 10 feet into metres. Then use the formula given in Question 1 above to find the perimeter of the lawn in metres.

(b) Use the formula given in Question 1 to find the perimeter of the lawn in feet, and then convert this into metres.

Check your answers

1 The length of the lawn is 10 m and its breadth is 8 m. So, from the formula,

Follow-up in Section 6.2.1.

$$\begin{aligned}\text{perimeter} &= (2\times\text{length}) + (2\times\text{breadth})\\ &= (2\times10\,\text{m}) + (2\times8\,\text{m}) = 20\,\text{m} + 16\,\text{m} = 36\,\text{m}.\end{aligned}$$

The perimeter of the lawn is 36 m.

Follow-up in Section 6.2.2. Note that ft is an abbreviation for foot or feet.

2 (a) As $1\,\text{ft} = 0.3048\,\text{m}$, it follows that

$$12.5\,\text{ft} = 12.5 \times 0.3048\,\text{m} = 3.810\,\text{m}$$

and

$$10\,\text{ft} = 10 \times 0.3048\,\text{m} = 3.048\,\text{m}.$$

Therefore the lawn is 3.810 m by 3.048 m.

From the formula for the perimeter,

$$\text{perimeter} = (2 \times 3.810\,\text{m}) + (2 \times 3.048\,\text{m}) = 13.716\,\text{m}.$$

So the perimeter of the lawn in metres is 13.716 m.

(b) From the formula for the perimeter,

$$\text{perimeter} = (2 \times 12.5\,\text{ft}) + (2 \times 10\,\text{ft}) = 45\,\text{ft}.$$

So the perimeter of the lawn in feet is 45 ft.

To convert this into metres, use $1\,\text{ft} = 0.3048\,\text{m}$,

then

$$45\,\text{ft} = 45 \times 0.3048\,\text{m} = 13.716\,\text{m}.$$

This is the same answer for the perimeter as in part (a). However, the method in part (b) is quicker and, as it involves fewer calculations, it affords less opportunity for making mistakes.

A further point to bear in mind is that the perimeter formula requires consistent units—you cannot, for instance, use length measured in feet and breadth measured in metres.

A **formula** is a rule or a generalization. Word formulas—formulas that use English words rather than mathematical symbols—are so much a part of life that people often use them without realizing that they are doing so. Here are some examples.

The cost of a purchase of oranges is the price per orange times the number of oranges.

The total cost of petrol is the price of petrol per litre times the number of litres.

The average speed is the distance travelled divided by the time taken.

The number of tins of paint needed to cover a wall is the area of the wall divided by the area that would be covered by one tin of paint.

All of these word formulas can be written more clearly using mathematical symbols as below; in particular, notice how brackets can be used to provide greater clarity.

Cost of oranges bought = (price per orange) × (number of oranges).

Cost of petrol = (price of petrol per litre) × (number of litres).

Average speed = (distance travelled)/(time taken).

Number of tins of paint needed = (area of wall)/(area covered by one tin).

6.2.1 Using word formulas

Formulas are important because they describe *general* relationships, rather than *specific* numerical ones. For example, the tins of paint formula applies to *every* wall. To use such a formula you need to substitute specific values for the general terms, as the following examples show.

Example 6

If the price of petrol is 85 pence per litre and a driver buys 25.9 litres, what is the cost?

Solution

The formula is

cost of petrol = (price of petrol per litre) × (number of litres).

The price per litre is 85 pence and the number of litres is 25.9. So

cost of petrol = (85 pence) × 25.9 = 2201.5 pence.

Rounded to the nearest penny, this is 2202 pence or £22.02.

Example 7

Suppose you are travelling on a UK motorway in a coach and you want to know how far the coach has travelled in the previous quarter of an hour. You glance over the shoulder of the driver and you see that the speed is a steady 70 miles per hour. Estimate how far you have travelled on the motorway in the last quarter of an hour, assuming that an average speed of 70 miles per hour has been maintained.

Solution

The formula gives

$$\begin{aligned}\text{distance travelled} &= (\text{average speed}) \times (\text{time taken})\\ &= (70\,\text{miles per hour}) \times (0.25\text{ hour})\\ &= 70 \times 0.25\,\text{miles} = 17.5\,\text{miles}.\end{aligned}$$

As already noted, the units used in a formula must be consistent. For instance, in the example above, the average speed is in *miles* per *hour*, so the time travelled must be in *hours* and the distance travelled must be in *miles*. If one of the quantities is not in the correct units, then you need to change it before using the formula.

Example 8

Suppose you have been travelling in a car at an average speed of 50 miles per hour for 45 minutes. How far would you have travelled?

Solution

You need to change the time into hours in order for the units to be consistent. Thus, 45 minutes is $\frac{3}{4}$ hour. Then, from the formula,

$$\begin{aligned} \text{distance travelled} &= (\text{average speed}) \times (\text{time taken}) \\ &= (50 \text{ miles per hour}) \times (\tfrac{3}{4} \text{ hour}) = 37.5 \text{ miles.} \end{aligned}$$

Solutions on page 108.

Try some yourself (6.2.1)

1 If tomatoes cost 75 pence per kg, how much would 1.45 kg cost?

2 If you travel in a car at an average speed of 60 km per hour, how far would you have travelled after 1.5 hours, after 2 hours 40 minutes and after three and a half hours?

3 You are planning to paint three rooms with total wall areas of 56, 38 and 40 square metres, using paint that comes in tins which claim to cover 15 square metres per tin. How many tins will you need for each room? And how many in total?

6.2.2 Converting units by using a formula

It is sometimes necessary to convert from one unit of measurement to another. You can use a formula in order to do this.

Example 9

Preparatory Handbook Activity Sheet 1 lists this and similar unit conversions.

Use the formula 1 pint = 0.5679 litre to calculate the number of litres in 8 pints (or 1 gallon).

Solution

As 1 pint is 0.5679 litre, it follows that 8 pints will be 8 times this:

$$8 \text{ pints} = 8 \times (0.5679 \text{ litre}) = 4.5432 \text{ litres.}$$

Sometimes the conversion formula that is given is in the opposite direction from the one you want. However, it is possible to turn such formulas round. For example, since $1\,\text{km} = 1000\,\text{m}$, it follows that $1\,\text{m} = \frac{1}{1000}\,\text{km} = 0.001\,\text{km}$.

Example 10

Turn the formula 1 mile = 1.609 km round to obtain 1 km in miles.

Solution

The formula can be written as

$$1.609\,\text{km} = 1\,\text{mile}.$$

Dividing by 1.609 gives

$$1\,\text{km} = \frac{1}{1.609}\,\text{mile} = 0.6215\,\text{mile (to 4 d.p.)}.$$

Try some yourself (6.2.2)

Solutions on page 108.

1 Use the formula 1 mile = 1.609 km to find 12 miles in kilometres.

2 Turn the following formulas round to obtain centimetres in terms of inches, and grams in terms of pounds:

$$1\,\text{inch} = 2.54\,\text{cm};$$
$$1\,\text{lb} = 453.6\,\text{g}.$$

How many pounds are there in a kilogram?

Outcomes

Now that you have studied Module 6 you should be able to:

- ◇ lay out and, where appropriate, label simple mathematical arguments;
- ◇ understand the precise mathematical meaning of certain common English words;
- ◇ understand and use common mathematical symbols;
- ◇ use word formulas;
- ◇ use formulas to convert units.

Module 7 Geometry

This module looks at various aspects of shape and space. It uses a lot of mathematical vocabulary, so you should make sure that you are clear about the precise meaning of words such as circumference, parallel, similar and cross-section.

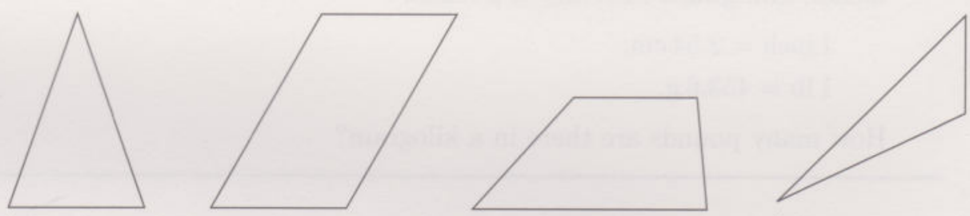
7.1 *Shapes and symmetry*

This section deals with the simplest geometric shapes and their symmetries. All of the shapes are two-dimensional—hence they can be drawn accurately on paper.

These are diagnostic questions. Try them to see which topics you need to revise.

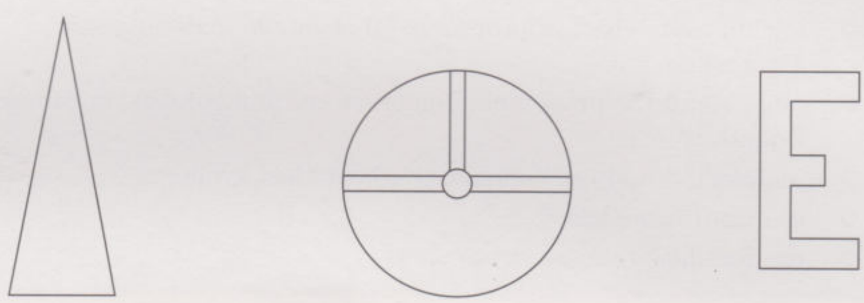
Try these first

1 Name the following shapes:



(a) (b) (c) (d)

2 Draw a line of symmetry on each of the shapes below.



(a) (b) (c)

3 Which of these have rotational symmetry?

(a) (b) (c)

Check your answers

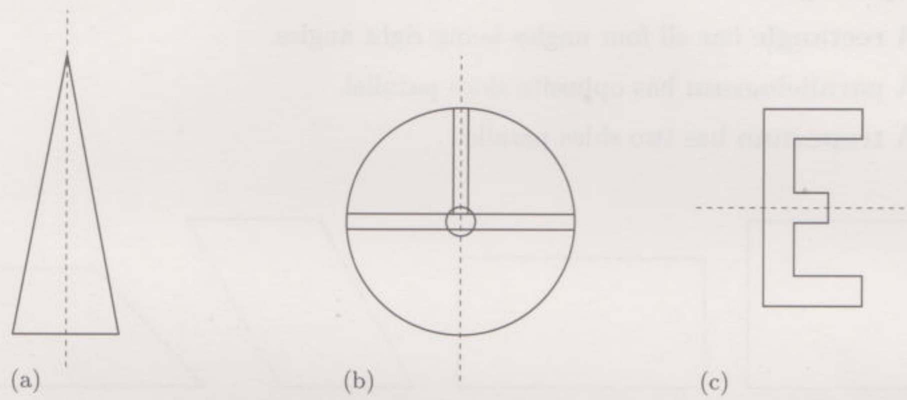
1 (a) Isosceles triangle (b) Parallelogram

(c) Trapezium (d) Scalene triangle

Follow-up in Section 7.1.1.

2 Each of the shapes has only one line of symmetry, so these are the only possible answers.

Follow-up in Section 7.1.2.



3 (a) The dartboard has rotational symmetry.

(b) The letter Z has rotational symmetry.

(c) The letter K does not have rotational symmetry.

Follow-up in Section 7.1.2.

7.1.1 Geometric shapes

Simple geometric shapes are studied in mathematics partly because they are used in thousands of practical applications. For instance, triangles occur in bridges, pylons and, more mundanely, in folding chairs; rectangles occur in windows, cinema screens and sheets of paper; while circles are an essential part of wheels, gears and plates.

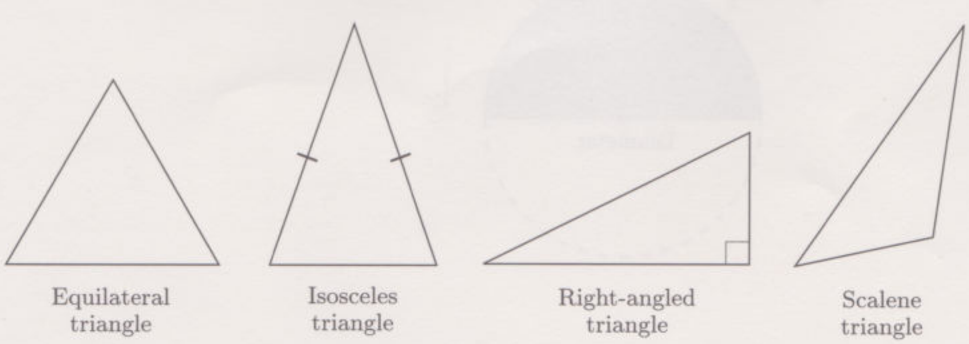
By definition, triangles are shapes with three straight sides. However, there are various types of triangle:

An **equilateral triangle** is a triangle with all three sides of equal length. The three angles are also all equal.

An **isosceles triangle** is a triangle with two sides of equal length. The two angles opposite the equal sides are also equal to one another.

A **right-angled triangle** is a triangle with one angle that is a right angle.

A **scalene triangle** is a triangle with all the sides of different lengths. The angles are also all different.



It is a general convention that equal sides are marked by drawing a short line, /, through them, and a right angle is marked by a square between the arms of the angle. This can be seen in the preceding diagram.

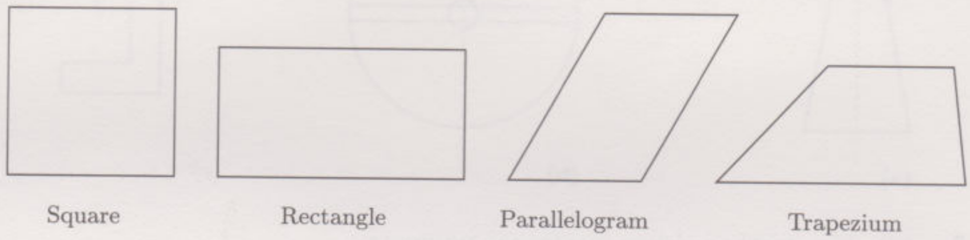
Quadrilaterals are shapes with four straight sides. Again, there are different types:

A **square** has all four sides the same length, with all four angles being right angles.

A **rectangle** has all four angles being right angles.

A **parallelogram** has opposite sides parallel.

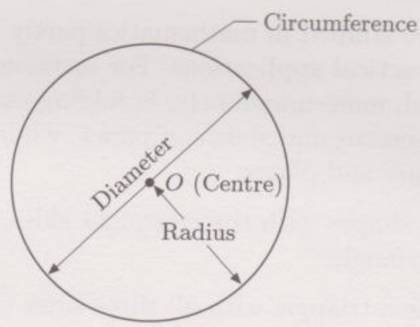
A **trapezium** has two sides parallel.



Next consider circles. All circles are the same shape—they can only have different sizes.

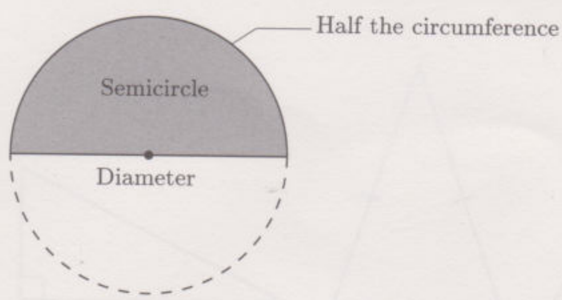
The centre is often labelled with the letter O.

In a circle, all the points are the same distance from a point called the centre.

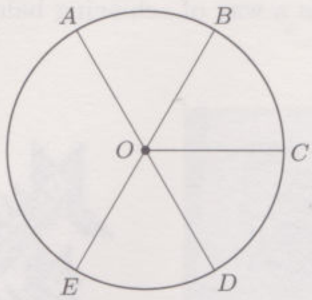


The plural of radius is radii.

The outside edge of a circle is called the **circumference**. A straight line from the centre to a point on the circumference is called a **radius** of the circle, and a line with both ends on the circumference and passing through the centre is called a **diameter**. Any diameter cuts the circle into two halves called **semicircles**.



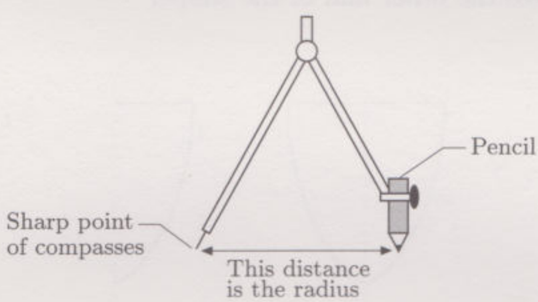
In the circle below, the lines labelled OA, OB, OC, OD and OE are all radii, and AD and BE are diameters. The points A, B, C, D and E all lie on the circumference.



Although the terms 'radius', 'diameter' and 'circumference' each denote a certain line, these words are also employed to mean the *lengths* of those lines. So it is common to say, for example, 'Mark a point on the circumference' and 'The circumference of this circle is 7.3 cm'. It is obvious from the context whether the line itself or the length is being referred to.

Drawing circles freehand often produces very uncircle-like shapes! If you need a reasonable circle, you could draw round a circular object, but if you need to draw an accurate circle with a particular radius, you will need a pair of compasses and a ruler. Using the ruler, set the distance between the point of the compasses and the tip of the pencil at the desired radius; place the point on the paper at the position where you want the centre of the circle to be and carefully rotate the compasses on the point so that the pencil marks out the required circle.

This single instrument is called a *pair* of compasses.



To draw a large circle, perhaps to create a circular flowerbed, a similar set-up is needed. The essentials are a fixed central point (possibly a stake) and a means of ensuring a constant radius (possibly a string). To draw a circle on a computer or calculator screen, you may also need to fix the centre (maybe using coordinates) and the radius.

It is often necessary to label diagrams of geometric figures, such as circles or triangles, in order to make it easier to refer to specific parts of the figure. A simple way would be to colour the lines, and so identify, say 'the red line' or 'the green circle', but that is not possible in this book as it does not use colour printing. For the most part, MU120 employs the age-old device of labelling points as A, B, C, ... and lines as AB, BC, ..., or a, b, c, ... and using combinations of the letters, such as 'triangle ABC'. It is rather laborious to read, but unfortunately is unavoidable.

'Triangle ABC' is often written as '$\triangle ABC$'.

Note that, as in the case of words like 'radius' and 'circumference', AB may be used to mean the line from A to B or the length of the line itself.

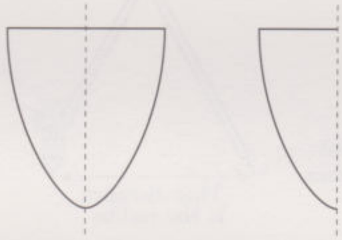
7.1.2 Symmetry

Symmetry is a feature that has been used in the design of objects and patterns in many cultures throughout recorded history. From Greek vases and medieval windows to Victorian tiles and Native American decorations, symmetry has been seen as a way of achieving balance and beauty.

Symmetry can be described mathematically, and is a useful concept when dealing with shapes.

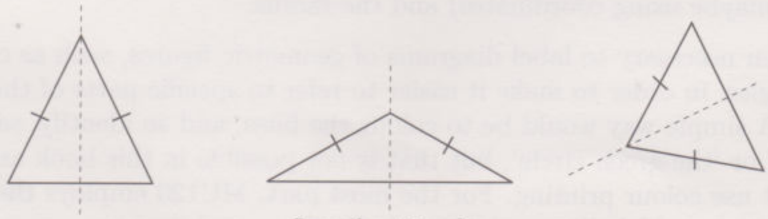
Look at the shapes below. The symmetry of the shape on the left and its relationship to the shape on the right can be thought of in two ways:

- Fold the left-hand shape along the central line. Then one side lies exactly on top of the other, and gives the shape on the right.
- Imagine a mirror placed along the central dotted line. The reflection in the mirror gives the other half of the shape.



This type of symmetry is called **line symmetry**.

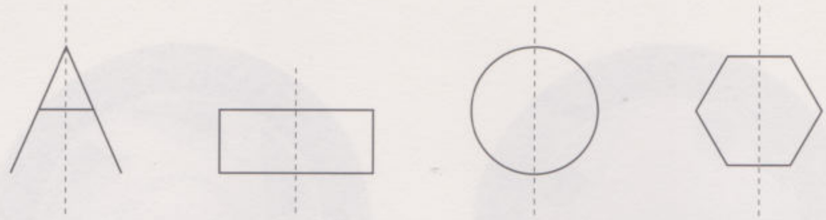
Any isosceles triangle has line symmetry.



Isosceles triangles

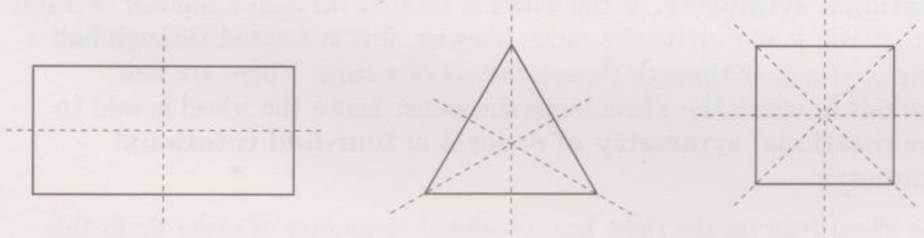
The dashed lines represent *lines of symmetry*, and each shape is said to be **symmetrical** about this line.

The following all have line symmetry:



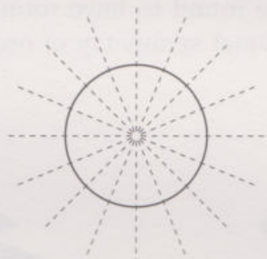
Letter A Top of a box Base of a vase Top of a bolt

A shape can have more than one line of symmetry. Thus a rectangle has two lines of symmetry, an equilateral triangle has three lines of symmetry, and a square has four.

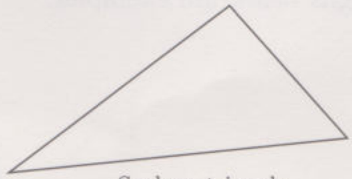


Rectangle Equilateral triangle Square

A circle has an infinite number of lines of symmetry since it can be folded about any diameter. Only eight of the possible lines of symmetry are indicated below.

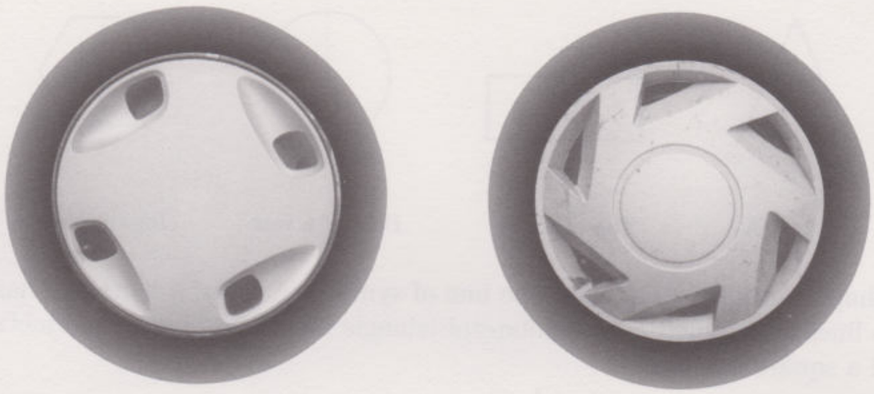


Some shapes, such as a scalene triangle, have no lines of symmetry—it is not possible to fold the shape about a line so that the two halves fit exactly on top of one another.



Scalene triangle

There is another kind of symmetry which is often used in designs. It can be seen, for instance, in car wheel trims.



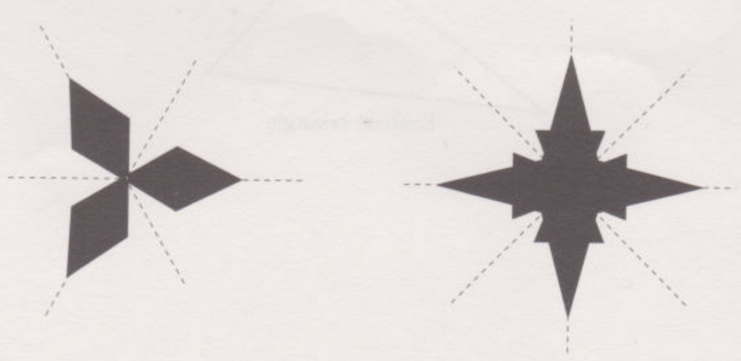
Look at the trim on the left. It does not have line symmetry but it has **rotational symmetry**. If the wheel is rotated through a quarter of a full turn, it will look exactly the same; likewise, if it is rotated through half a complete turn, or through three-quarters of a turn. There are four positions in which the wheel looks the same: hence the wheel is said to have **rotational symmetry of order 4** or **four-fold rotational symmetry**.

The wheel trim on the right has rotational symmetry of order 6. In this case there are six positions in which the trim will look exactly the same. These occur when the wheel is rotated through one-sixth of a complete turn, two-sixths of a turn, and so on, to five-sixths of a turn and finally a complete turn (when, of course, the wheel is back in its original position).

The centre of the shape is the point about which the shape is rotated; it is called **the centre of rotation**.

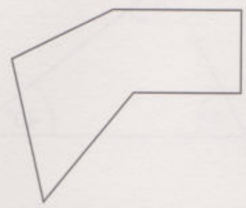
A shape does not have to be round to have rotational symmetry. The following shapes have rotational symmetry of orders 3 and 4, respectively.

It is not difficult to create shapes with both line symmetry and rotational symmetry. The two designs below are examples.



The design on the left has three lines of symmetry and rotational symmetry of order 3. The one on the right has four lines of symmetry and rotational symmetry of order 4.

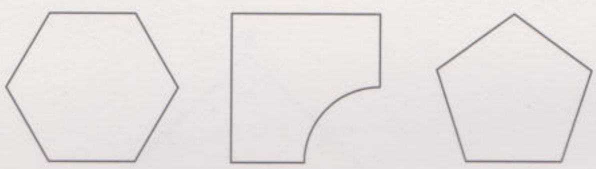
A shape with no rotational symmetry, like the one below, is sometimes said to have 'rotational symmetry of order 1'. This is because it will only fit on top of itself in one position—after a complete turn.



Try some yourself (7.1.2)

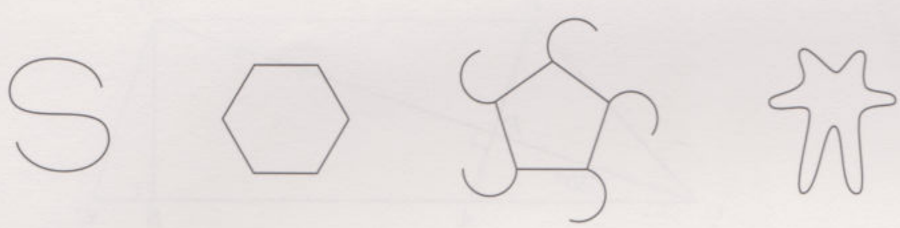
Solutions on page 108.

1 Mark all of the lines of symmetry on these shapes. For each shape, state the total number of lines of symmetry.



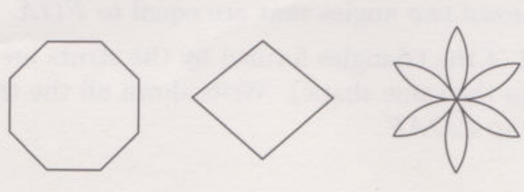
(a) (b) (c)

2 Mark the centre of rotation on each of the shapes below. For each, state the order of rotational symmetry.



(a) (b) (c) (d)

3 Describe the symmetry of each of these shapes. Mark all the lines of symmetry in each case. Also mark the centre of rotation, and state the order of rotational symmetry.



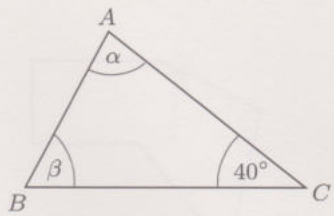
(a) (b) (c)

7.2 Angles

These are diagnostic questions. Try them to see which topics you need to revise.

Try these first

1



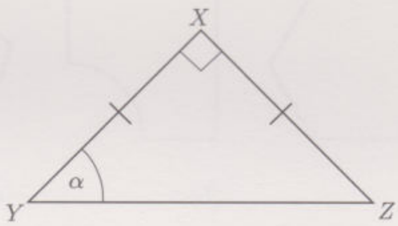
(a) How would you refer to angle α in this triangle by means of the letters A, B and C?

(b) Measure α with a protractor.

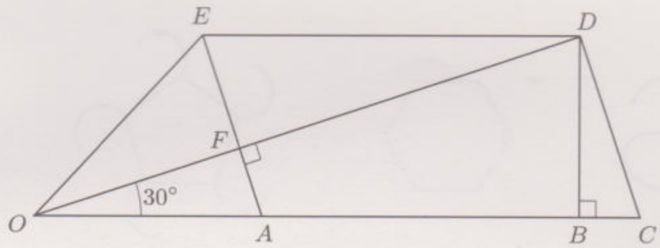
(c) What type of angle is α?

(d) Find β without using a protractor.

2 Deduce the value of α in the triangle below.



3 This diagram shows the arrangement of the struts in a wall of a shed.



The lines $OABC$ and DE are each horizontal. The struts EA and DC are parallel.

(a) Which of these are right angles?

$O\widehat{F}A$, $E\widehat{F}D$, $E\widehat{D}B$, $O\widehat{D}C$.

(b) Write down two angles that are equal to $F\widehat{O}A$.

(c) Several of the triangles formed by the struts are similar (that is, they are the same shape). Write down all the triangles that are similar to $\triangle OAF$.

Check your answers

1 (a) Any of the following could be used: $\widehat{A}$, $B\widehat{A}C$, $\angle BAC$, $C\widehat{A}B$, $\angle CAB$. — Follow-up in Section 7.2.1.

(b) $\alpha = 80°$.

(c) Because α is less than 90°, it is an acute angle.

(d) As the three angles of a triangle always add up to 180°,

$$\beta + 80° + 40° = 180°.$$

Therefore

$$\beta = 180° - 120° = 60°.$$

2 Lines XY and XZ are of equal length. This means that the triangle is isosceles, so the base angles $\widehat{Y}$ and $\widehat{Z}$ are equal. Then $\widehat{Y} = \widehat{Z} = \alpha$. The third angle in the triangle is a right angle: $\widehat{X} = 90°$. — Follow-up in Section 7.2.3.

Because the three angles in a triangle must add up to 180°,

$$\alpha + \alpha + 90° = 180°.$$

Hence

$$2\alpha = 180° - 90° = 90°$$

and

$$\alpha = 45°.$$

3 (a) All four of the given angles are right angles. — Follow-up in Sections 7.2.2 and 7.2.4.

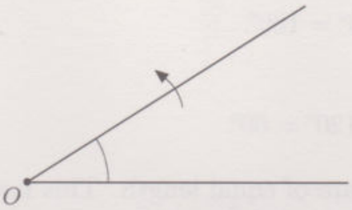
(b) $E\widehat{D}F$ (which is the same as $F\widehat{D}E$) is equal to $F\widehat{O}A$. They are alternate angles.

$C\widehat{D}B$ (which is the same as $B\widehat{D}C$) is equal to $F\widehat{O}A$. This is because $\triangle BCD$ and $\triangle OAF$ are similar: each has a right angle, and $B\widehat{C}D$ and $O\widehat{A}F$ are corresponding angles.

(c) There are four triangles that are similar to $\triangle OAF$: they are $\triangle OBD$, $\triangle DEF$, $\triangle OCD$ and $\triangle BCD$.

7.2.1 Angles, notation and measurement

In everyday language, the word 'angle' is often used to mean the space between two lines ('The two roads met at a sharp angle') or a rotation ('Turn the wheel through a large angle'). Both of these senses are used in mathematics, but it is probably easier to start by thinking of an angle in terms of the second of these—as a rotation.



The diagram above shows a fixed arm and a rotating arm (with the arrow), which are joined together at O, forming an angle between them. Imagine that the rotating arm, which is pivoted at O, initially rests on top of the fixed arm, and that it then rotates in the direction of the arrow. Focus on the size of the marked angle between the arms. At first the angle is quite sharp, but it becomes less so. It then becomes a right angle, and subsequently gets much blunter until the two arms form a straight line. Thereafter it starts to turn back upon itself, passing through a three-quarter turn and, when the rotating arm gets back to the start, it rests on top of the fixed arm again.

Most of these angles have names, as summarized below. They can also be measured: the most common unit for expressing angles is degrees, denoted by °, with a complete turn or revolution being equal to 360°.

Angles can also be measured in *radians*, and you will meet this unit of measure later in MU120.

Acute angle — Any angle that is less than a quarter turn; that is, less than 90°. An example of an acute angle is the angle that a door makes with a doorframe when it is ajar.

Right angle — The angle that corresponds to a quarter turn; it is exactly 90°. The angles at the corners of most doors, books and windows are right angles.

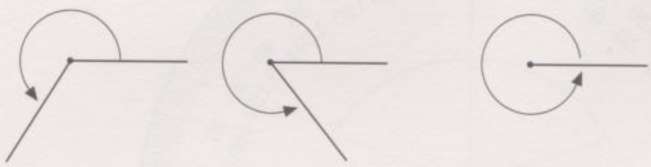
Obtuse angle — Any angle that is between a quarter turn and a half turn; that is, between 90° and 180°. An example is the angle between the blades of a pair of scissors when they are open as wide as possible.

Half turn (Straight angle) — This corresponds to a straight line; it is exactly 180°. The pages of an open book that is lying flat approximately describe a half turn.

Reflex angle — Any angle that is between a half turn and a complete turn; that is, between 180° and 360°. When a box is opened and the hinged lid falls back so as to rest on the surface on which the box is standing, the angle that the lid turns through is a reflex angle.

Complete turn — This corresponds to a complete turn, or one revolution; it is exactly 360°. This is the angle that the minute hand of a clock turns through in an hour.

Acute angle | Right angle 90° Quarter turn | Obtuse angle | Straight angle 180° Half turn



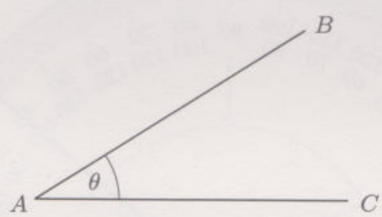
Reflex angles | One complete turn 360°

Remember that if the angle between two straight lines is 90°, then the lines are said to be **perpendicular** to each other.

Sometimes it is necessary to refer to a turn that is more than one complete revolution, and so is greater than 360°. An example is the angle that the minute hand of a clock turns through in a period of 12 hours: each complete revolution of the minute hand amounts to 360°, so twelve revolutions amount to $12 \times 360° = 4320°$.

Several different notations are used for angles. Very often Greek letters are employed, as here.

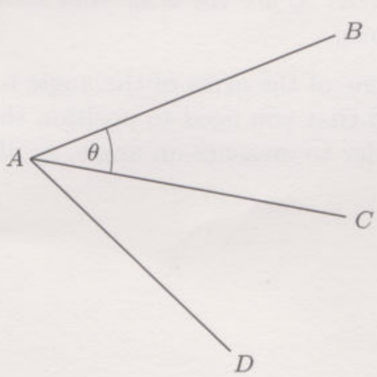
The Greek alphabet is listed on Preparatory Handbook Activity Sheet 4.



Alternatively, an angle may be denoted by the label on the vertex but with a hat on it. For instance, the angle θ above may be denoted by $\widehat{A}$, which is read as 'angle A'.

The vertex is another name for the 'corner' of an angle.

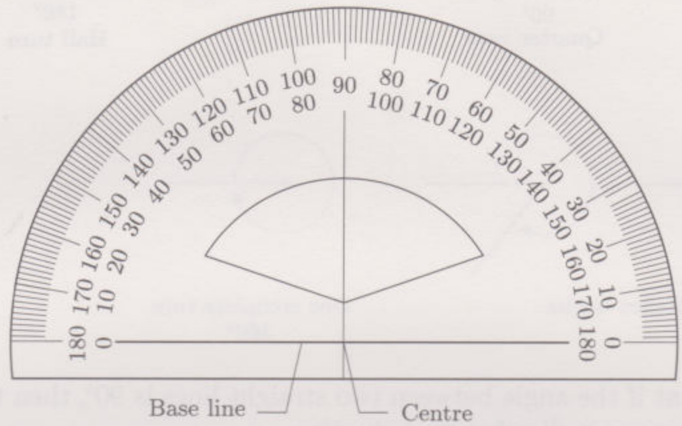
This notation can be ambiguous if there is more than one angle at the vertex, as in the example below.



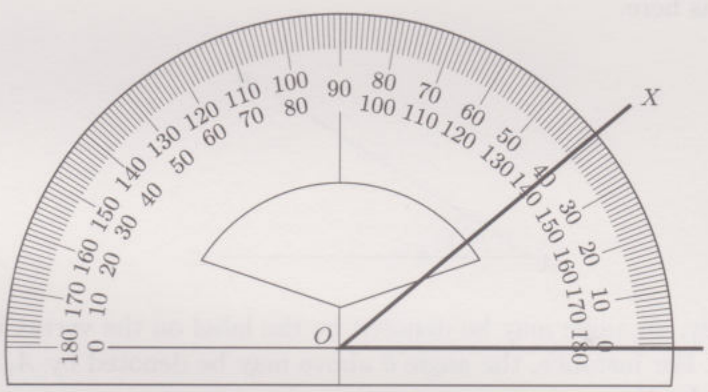
In such cases, θ can be specified as $C\widehat{A}B$, $B\widehat{A}C$, $\angle CAB$ or $\angle BAC$—the middle letter indicates the vertex and the two outer letters identify the 'arms' of the angle.

Both $C\widehat{A}B$ and $\angle CAB$ are read as 'angle CAB'.

To measure an angle you need a protractor. The protractor shown here is a semicircle that is graduated to measure angles from 0° to 180°. It is also possible to buy circular protractors that measure angles from 0° to 360°.

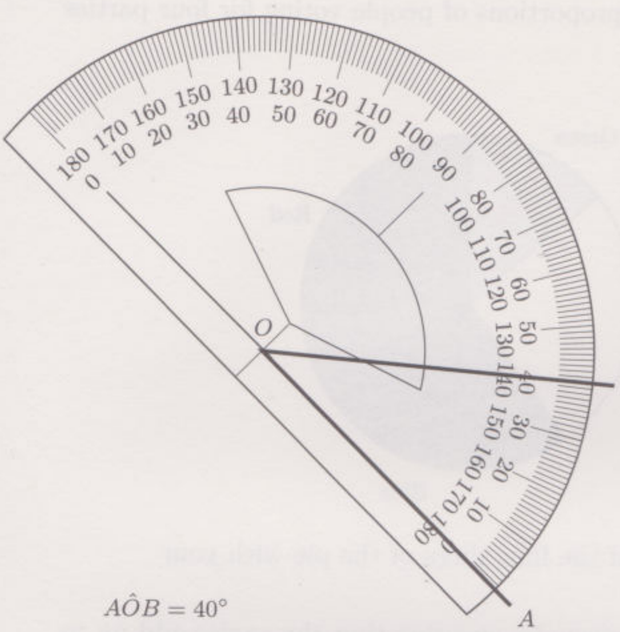


The diagram below indicates how the protractor should be positioned in order to measure an angle. Place the base line of the protractor on one arm of the angle, with the centre O on the vertex. The angle can then be read straight from the scale. Here $Y\widehat{O}X = 40°$ (*not* 140°).

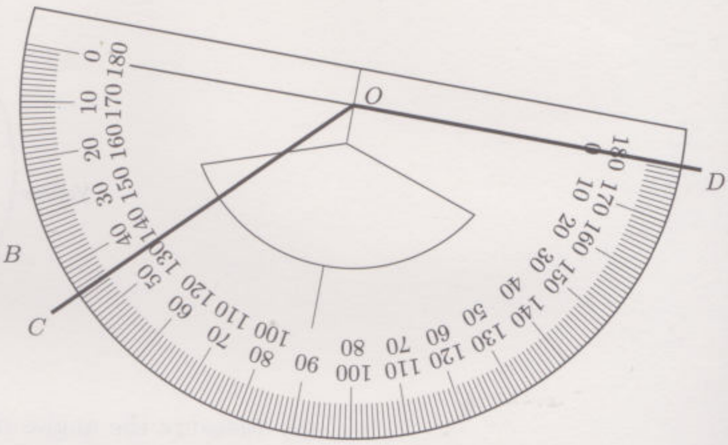


Be careful to use the correct scale. In this case the angle extends from the line OY up to the line OX, so use the scale that shows OY as 0°—the outer scale in this instance.

In the above example, one of the arms of the angle is horizontal. However, sometimes you may find that you need to position the protractor in an awkward position in order to measure an angle, as illustrated on the opposite page.



$A\hat{O}B = 40°$



$C\hat{O}D = 135°$

You can also use a protractor to construct an angle accurately, but once you have drawn the angle, be on the safe side and measure it to check that it is correct.

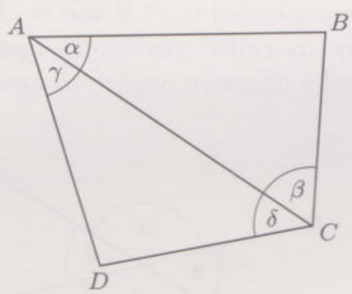
Try some yourself (7.2.1)

Solutions on page 109.

1 What angles do the hour hand and the minute hand of a clock turn through in five hours?

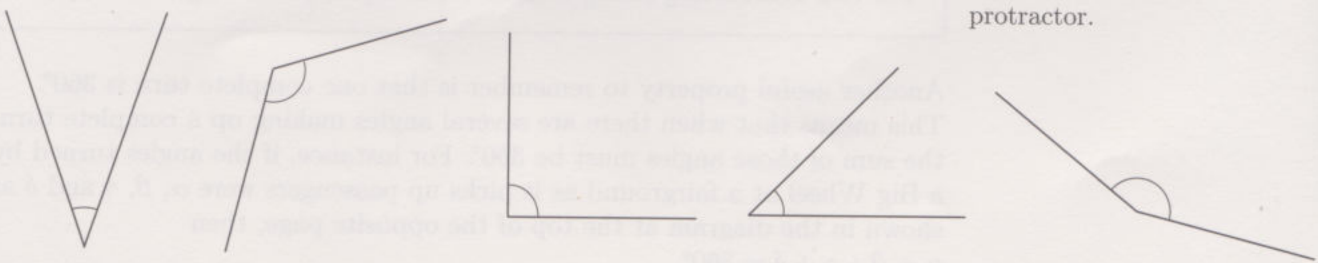
2 Give an alternative notation for labelling each of these angles in the diagram below.

(a) α (b) β (c) $D\widehat{A}C$ (d) $\angle ACD$



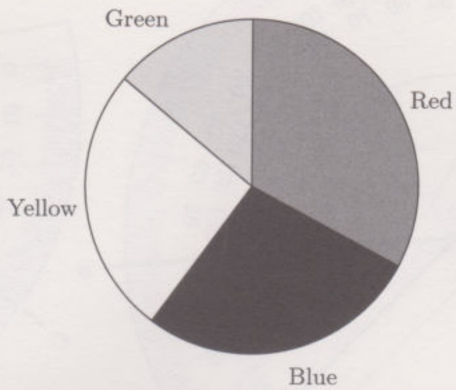
3 Use your protractor to measure each of the marked angles, and state whether the angle is acute, right, obtuse, etc.

Note that it will be necessary to extend the arms of the angles so that they overlap with the scale on the protractor.



(a) (b) (c) (d) (e)

4 This pie chart shows the proportions of people voting for four parties in a local election.



(a) Measure the angles of the four slices of the pie with your protractor.

(b) Check your measurements by ensuring that the angles add up to 360°.

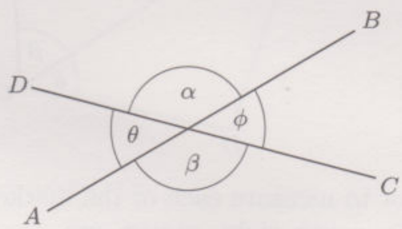
(c) Work out the percentage of the total vote polled by each of the four parties.

7.2.2 *Angles, points and lines*

Very often, angles in a shape are determined by the geometric properties of that shape. For example, a square has four right angles. So, when you know a shape is a square, you do not need to *measure* its angles to know that they are 90°. The rest of this section will look at the properties of shapes that enable you to deduce and calculate angles rather than measure them.

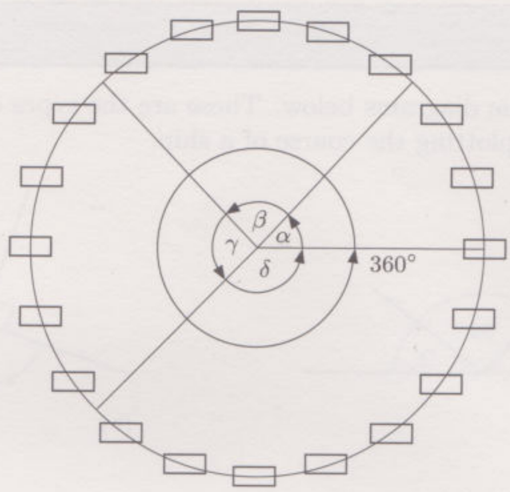
When two straight lines cross, they form four angles. In the diagram below, these angles are labelled α, β, θ and ϕ. The angles opposite each other are equal. They are called **vertically opposite** angles. Here α and β are a pair of vertically opposite angles, as are θ and ϕ.

Although such angles are called 'vertically opposite', they do not need to be vertically above and below each other!



For two intersecting straight lines, vertically opposite angles are equal.

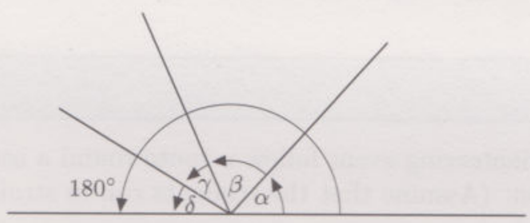
Another useful property to remember is that one complete turn is 360°. This means that when there are several angles making up a complete turn, the sum of those angles must be 360°. For instance, if the angles turned by a Big Wheel at a fairground as it picks up passengers were α, β, γ and δ as shown in the diagram at the top of the opposite page, then $\alpha + \beta + \gamma + \delta = 360°$.



The sum of angles at a point is 360°.

Similarly, if several angles make up a half turn, then the sum of those angles must be $\frac{1}{2} \times 360° = 180°$. Therefore, in the following diagram, $\alpha + \beta + \gamma + \delta = 180°$.

Note that different diagrams can be labelled with the same letters (α, β, γ and δ in this case). The letters represent different angles here to those in the preceding diagram.



The sum of angles on a line is 180°.

You can sometimes use these properties to determine unknown angles.

Example 1

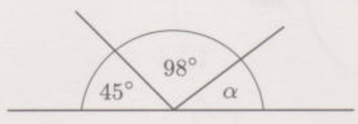
Calculate the angle between adjacent spokes of this wheel.

Solution

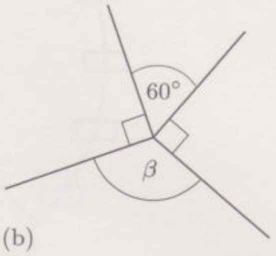
The eight spokes divide the circle up into eight equal parts. Therefore the angle required is found by dividing 360° by 8 to give 45°.

Example 2

Find α and β in the diagrams below. These are the types of diagram that might arise when plotting the course of a ship.



(a)



(b)

Solution

(a) As the angles are on a line,

$$45^\circ + 98^\circ + \alpha = 180^\circ,$$

then $\alpha = 180^\circ - 45^\circ - 98^\circ = 37^\circ$.

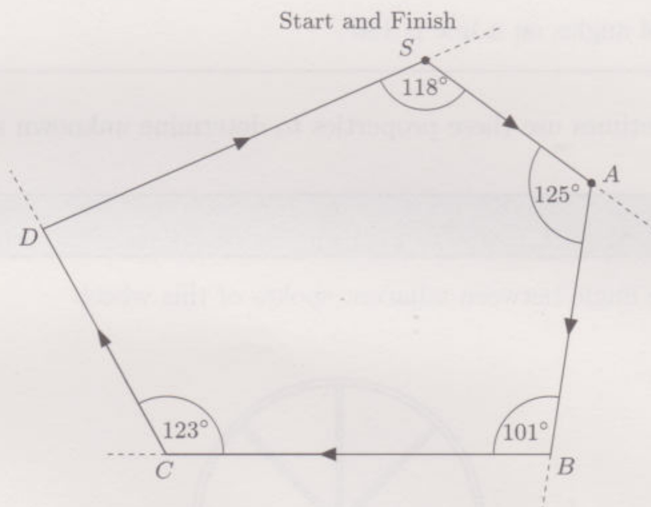
(b) As the angles are at a point,

$$90^\circ + 60^\circ + 90^\circ + \beta = 360^\circ,$$

then $\beta = 360^\circ - 90^\circ - 60^\circ - 90^\circ = 120^\circ$.

Example 3

Students at an orienteering event follow a route round a set course in a clockwise direction. (Assume that the students run in straight lines and keep to the track.)



(a) Through what angle do the students turn at A?

(b) When they arrive at D, what is the *total* angle that they have turned through relative to their starting direction?

(c) When they return to S, through what angle must they turn in order to face in the direction in which they started?

(d) When they reach D, through what angle must they turn in order to return to the start?

Solution

(a) The angle turned through at A is $180° - 125° = 55°$.

(b) The angle turned through at B is $180° - 101° = 79°$, and the angle turned through at C is $180° - 123° = 57°$.

So the total angle that the students have turned through when they arrive at D is $55° + 79° + 57° = 191°$.

(c) The angle that the students need to turn through at S is $180° - 118° = 62°$.

(d) Suppose the students complete the whole course and, at the finish, face in the same direction as at the start, they will overall have made one complete turn, that is, 360°.

So

$$191° + 62° + \text{angle turned through at } D = 360°,$$

hence

$$\text{angle turned through at } D = 360° - 253° = 107°.$$

Example 4

Over a five-year period a mathematics tutor found that 16 of her students gained distinctions, 32 gained pass grades and 12 failed to complete the course. Draw a pie chart to represent these data.

Solution

First, calculate how many students there were altogether:

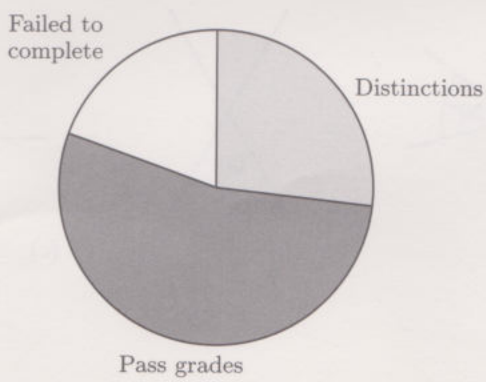
$$16 + 32 + 12 = 60 \text{ students.}$$

The whole pie chart (360°) must, therefore, represent 60 students. This means that each student is represented by $360° \div 60 = 6°$. So the angles for the three slices are

$$\begin{aligned} \text{distinctions} \quad & 16 \times 6 = 96°, \\ \text{pass grades} \quad & 32 \times 6 = 192°, \\ \text{failed to complete} \quad & 12 \times 6 = 72°. \end{aligned}$$

The pie chart can be constructed by carefully measuring these angles at the centre of a circle. The slices should be labelled, and an appropriate title given to the chart.

You may notice that not all charts and tables in this book have such titles. This is because the information is given immediately before the chart or table.



Pie chart showing results of 60 mathematics students over a five-year period

Solutions on page 109.

Try some yourself (7.2.2)

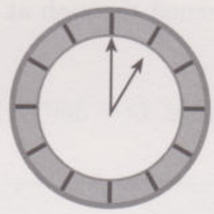
1 Calculate all the angles at the centres of these objects.

(a) Floor tiles

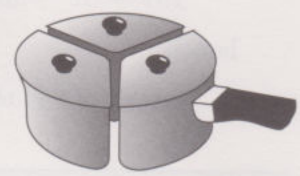
(b) Steering-wheel

(c) Needlework box

(d) Dart-board

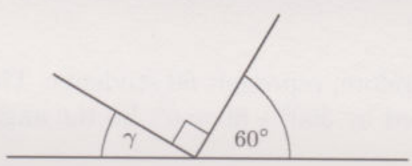


(e) Clock

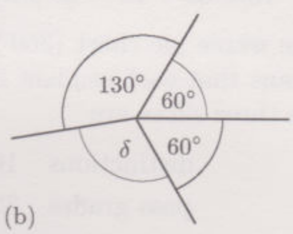


(f) Set of pans

2 Find γ and δ in the following diagrams produced by a ship's navigator.

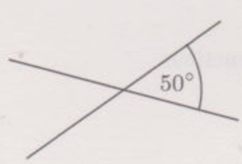


(a)



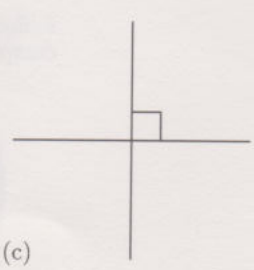
(b)

3 Find all the remaining angles in each of the diagrams below.



(a)

(b)



(c)

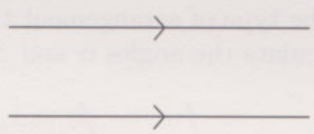
4 A company carried out a survey, recording how staff in a particular office spent their working time. The table shows the average number of minutes spent in each hour on various activities.

Activity	Time taken on average in one hour/mins
Keyboarding	35
Answering telephone	12
Talking with colleagues	10
Other	3

Draw a pie chart to represent these data.

7.2.3 Parallel lines

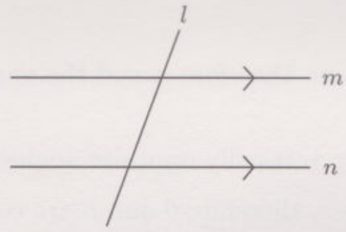
Two straight lines that do not intersect, no matter how far they are extended, are said to be **parallel**.



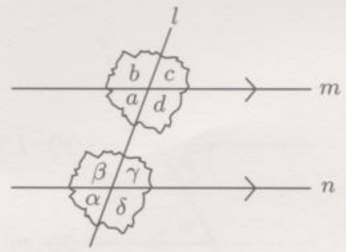
Arrows are used to indicate parallel lines.

When a pair of parallel lines is cut by another straight line, various pairs of equal angles are created. There are two important kinds of pairs—parallel lines make *corresponding angles* equal and *alternate angles* equal.

Look at the line l, which cuts two parallel lines m and n.

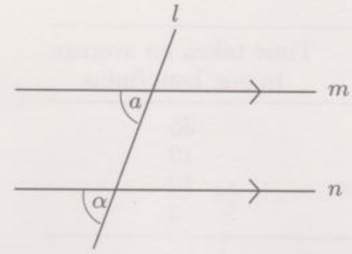


If you trace the lines at one of the intersections in the diagram above and place them over the lines at the other intersection, you will find that the two sets of lines coincide exactly. The four angles at each intersection also coincide exactly: thus $\alpha = a$, $\beta = b$, $\gamma = c$ and $\delta = d$.



The pairs of angles that correspond to each other at such intersections are called **corresponding angles**.

In the diagram below, α and a are corresponding angles: they are equal because m and n are parallel.

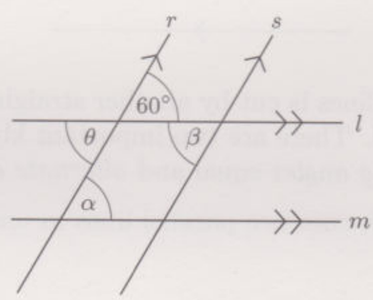


When a line intersects two parallel lines, corresponding angles are equal.

Example 5

This diagram represents the type of arrangement that occurs in a garden trellis or a wine rack. Calculate the angles α and β.

r and s are parallel lines, indicated by the single arrowheads; l and m are also parallel, indicated by the double arrows.



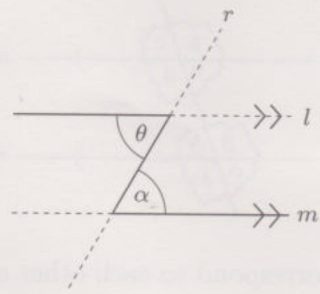
Solution

Line l is parallel to line m, therefore α and the angle 60° are corresponding angles. So $\alpha = 60°$.

The angles 60° and θ are vertically opposite angles. So $\theta = 60°$.

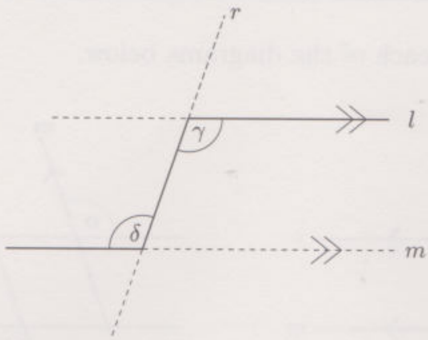
Line r is parallel to line s, therefore θ and β are corresponding angles. So $\beta = 60$.

Another pair of equal angles can be identified in Example 5. Notice that $\alpha = 60°$ and $\theta = 60°$. These two angles occur in a Z-shape, as indicated by the solid line in the diagram below. Such angles are called **alternate angles**.



When a line intersects two parallel lines, alternate angles are equal.

The other two angles are also equal and are also called alternate angles.

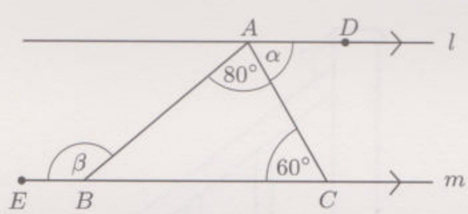


It is important to realize that you can find the sizes of unknown angles in many shapes by using a combination of the angle properties that have been outlined. To recap:

- Vertically opposite angles are equal.
- Angles at a point add up to 360°.
- Angles on a straight line add up to 180°.
- Corresponding angles on parallel lines are equal.
- Alternate angles on parallel lines are equal.

Example 6

Find α and β in the following diagram.



Solution

Line l is parallel to line m, therefore $C\widehat{A}D$ and $A\widehat{C}B$ are alternate angles. So

$$\alpha\,(C\widehat{A}D) = A\widehat{C}B = 60°.$$

Similarly, $A\widehat{B}E$ and $B\widehat{A}D$ are alternate angles. But

$$B\widehat{A}D = 80° + \alpha = 80° + 60° = 140°,$$

and hence

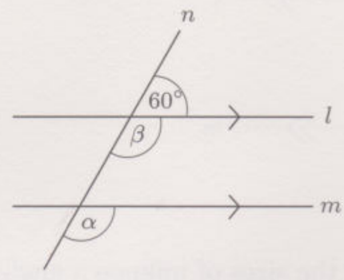
$$\beta\,(A\widehat{B}E) = B\widehat{A}D = 140°.$$

These properties of corresponding and alternate angles mean that the opposite angles in a parallelogram are also equal.

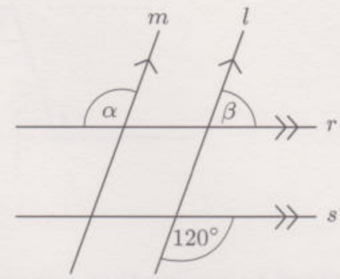
Solutions on page 110.

Try some yourself (7.2.3)

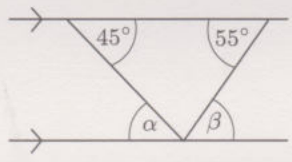
1 Find α and β in each of the diagrams below.



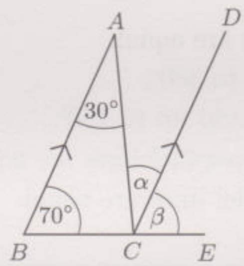
(a)



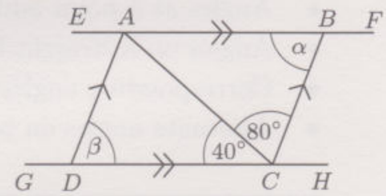
(b)



(c)

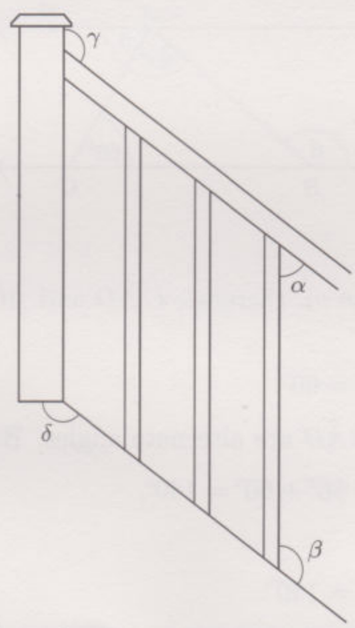


(d)

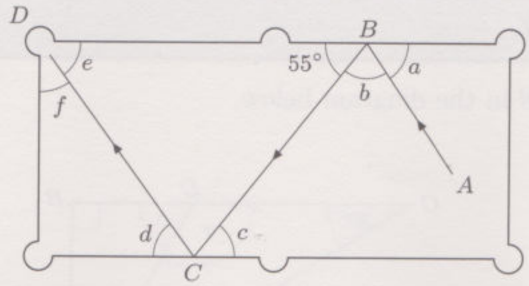


(e)

2 This diagram shows part of some bannister rails. The handrail makes an angle of 40° with the horizontal. Calculate angles α, β, γ and δ.

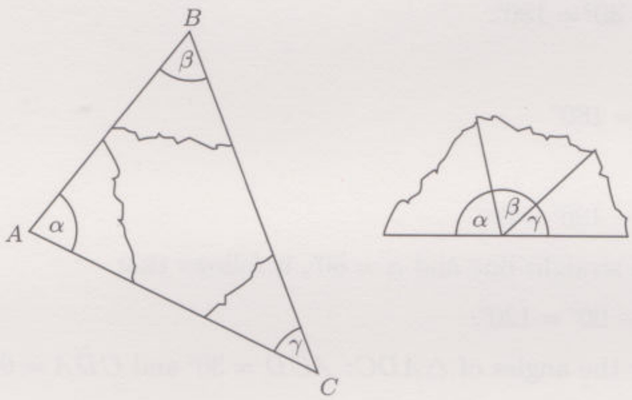


3 The arrows on the diagram at the top of the opposite page indicate the idealized path ($ABCD$) of a snooker ball on a snooker table. Assume that the angles between the cushion (the edge of the snooker table) and the path of the ball before and after it impacts with the cushion are equal. Calculate the sizes of the angles marked a, b, c, d, e and f.

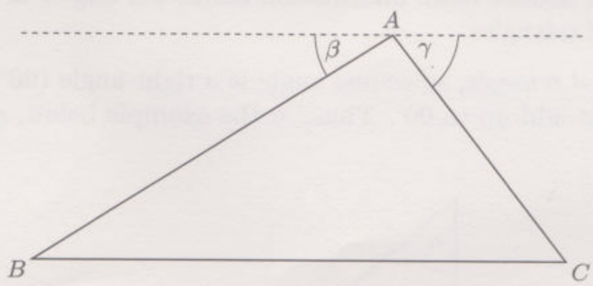


7.2.4 The angles of a triangle

The sum of the angles of any triangle is 180°. This property can be demonstrated in several ways. One way is to draw a triangle on a piece of paper, mark each angle with a different symbol, and then cut out the angles and arrange them side by side, touching one another as illustrated.



You can see *why* it is that the angles fit together in this way by looking at the triangle below. An extra line has been added parallel to the base. The angle of the triangle, $\widehat{B}$, is equal to the angle β at the top (they are alternate angles), and similarly the angle of the triangle, $\widehat{C}$, is equal to the angle γ at the top (they are also alternate angles). The three angles at the top (β, γ and the angle of the triangle, $\widehat{A}$) form a straight line of total angle 180°, and so the angles of the triangle must also add up to 180°.

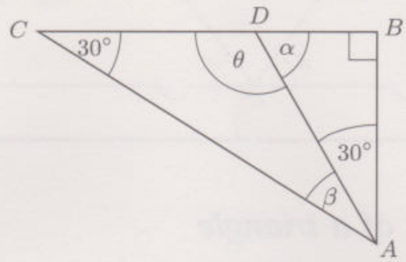


The sum of the angles of a triangle is 180°.

The fact that the angles of a triangle add up to 180° is another angle property that enables you to find unknown angles.

Example 7

Find α, β and θ in the diagram below.



Solution

First, look at the angles of $\triangle ABD$: $D\widehat{B}A = 90°$ and $B\widehat{A}D = 30°$.

Then, by the angle sum property of triangles,

$$\alpha + 90° + 30° = 180°.$$

So

$$\alpha + 120° = 180°$$

and

$$\alpha = 180° - 120° = 60°.$$

As CDB is a straight line and $\alpha = 60°$, it follows that

$$\theta = 180° - 60° = 120°.$$

Now consider the angles of $\triangle ADC$: $A\widehat{C}D = 30°$ and $C\widehat{D}A = \theta = 120°$.

Therefore

$$\beta + 30° + 120° = 180°.$$

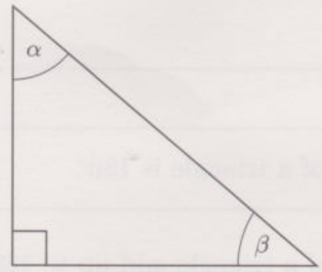
So

$$\beta = 180° - 150° = 30°.$$

(Check for yourself that the angles of $\triangle ABC$ also add up to 180°.)

It is possible to deduce more information about the angles in certain special kinds of triangles.

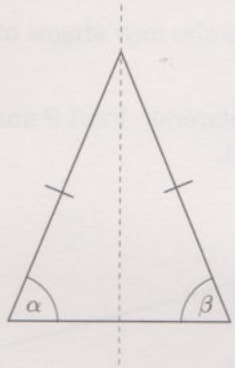
In a *right-angled triangle*, since one angle is a right angle (90°), the other two angles must add up to 90°. Thus, in the example below, $\alpha + \beta = 90°$.



In an *equilateral triangle*, all the angles are the same size. So each angle of an equilateral triangle must be $180° \div 3 = 60°$.

In an *isosceles triangle*, two sides are of equal length and the angles opposite those sides are equal. Therefore, $\alpha = \beta$ in the triangle below.

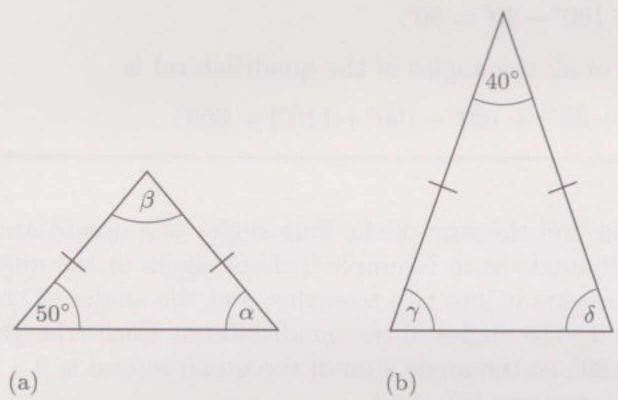
Such angles are often called **base angles**.



This means that there are only two different sizes of angle in an isosceles triangle: if the size of one angle is known, the sizes of the other two angles can easily be found. The next example shows how this is done.

Example 8

Find the unknown angles in these isosceles triangles, which represent parts of the roof supports of a house.



Solution

(a) As α and $50°$ are the base angles, $\alpha = 50°$. By the angle sum property of triangles,

$$\beta + 50° + 50° = 180°,$$

therefore

$$\beta = 180° - 50° - 50° = 80°.$$

(b) As γ and δ are the base angles, $\gamma = \delta$. In this triangle,

$$\gamma + \delta + 40° = 180°$$
$$\gamma + \delta = 180° - 40° = 140°,$$

therefore

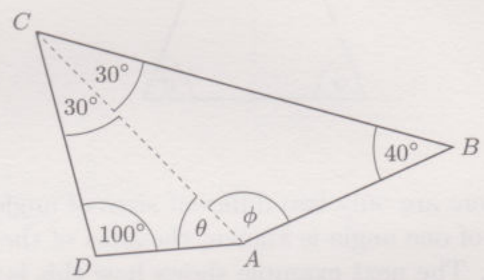
$$2\gamma = 140°$$
$$\gamma = 70°.$$

The various angle properties can also be used to find the sum of the angles of a quadrilateral.

Example 9

The diagram below represents the four stages of a walk drawn on an Ordnance Survey map.

The figure $ABCD$ is a quadrilateral. Find θ and ϕ, and thus the sum of all the angles of the quadrilateral.



Solution

From $\triangle ABC$,

$$\phi = 180° - 30° - 40° = 110°.$$

From $\triangle ACD$,

$$\theta = 180° - 100° - 30° = 50°.$$

Then the sum of all the angles of the quadrilateral is

$$40° + (30° + 30°) + 100° + (50° + 110°) = 360°.$$

In fact, you can find the sum of the four angles of a quadrilateral without calculating each angle as in Example 9. Look again at the quadrilateral: the dotted line splits it into two triangles, and the angles of these triangles together make up the angles of the quadrilateral. Each triangle has an angle sum of 180°, so the angle sum of the quadrilateral is $2 \times 180° = 360°$. This is true for *any* quadrilateral.

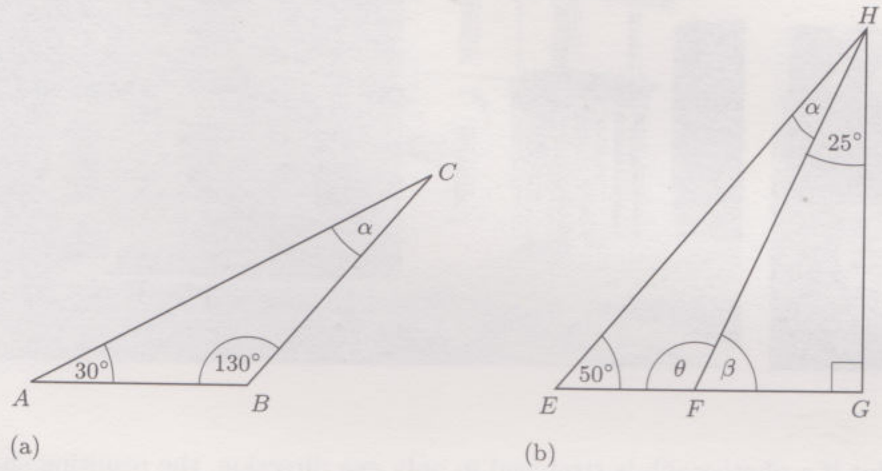
The sum of the angles of a quadrilateral is 360°.

Similarly, other polygons (that is, other shapes with straight sides) can be divided into triangles to find the sum of their angles.

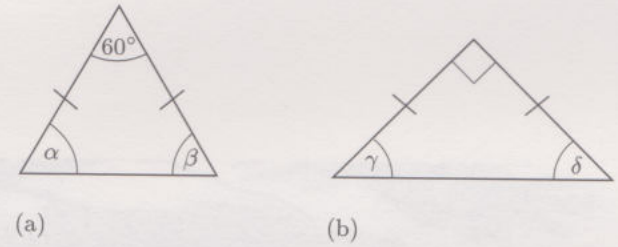
Try some yourself (7.2.4)

Solutions on page 111.

1 Find the unknown angles in each of these diagrams, which represent part of the bracing structure supporting a marquee.



2 Find the unknown angles in the following isosceles triangles, which represent roof rafters.



7.2.5 Similar and congruent shapes

Two shapes are said to be **similar** if they are the same shape but not necessarily the same size. In other words, one may be an enlargement of the other. They may also have different orientations, as in the drawing below.

When a photograph is enlarged, the two images are similar.

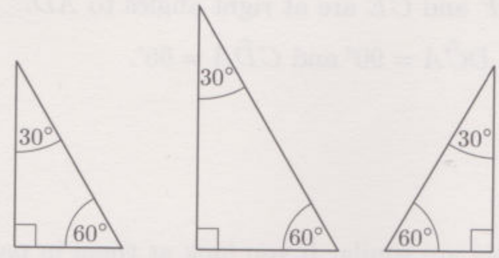
But if a photograph is stretched in only one direction, the resulting shape is not similar to the original.

In effect, when two shapes are similar, one is a scaled up (or down) version of the other. Thus an accurate model and its original will be similar in this mathematical sense. If you measure the sides of the model, you will find that to produce the original, each side must be scaled up by the same amount. However, the angles remain the same in each version.

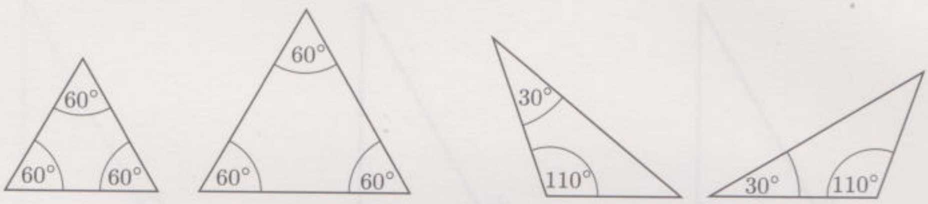
The simplest scaled shapes are similar triangles. In two similar triangles, angles in equivalent positions must be the same size. This provides a way of identifying similar triangles.

It is not necessary to calculate all the angles in two similar triangles. If two angles in one triangle match two angles in the other, then the third angle must also be the same in both, because in each case it will be 180° minus the sum of the other two angles.

Examples of similar triangles are set out below.



(a)

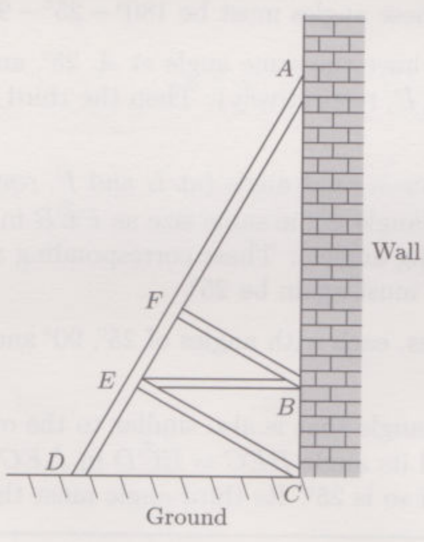


(b) (c)

If two figures are the same shape *and* the same size, they are said to be **congruent**.

Example 10

This diagram shows, in simplified form, a wooden buttress supporting the wall of a medieval church.



The angle between the ground and the buttress, $C\hat{D}A$, is 65°. By making appropriate assumptions, identify which triangles are similar. Calculate all the angles in the structure.

Solution

Assume that the wall is vertical, and that the ground and BE are both horizontal. Also assume that BF and CE are at right angles to AD.

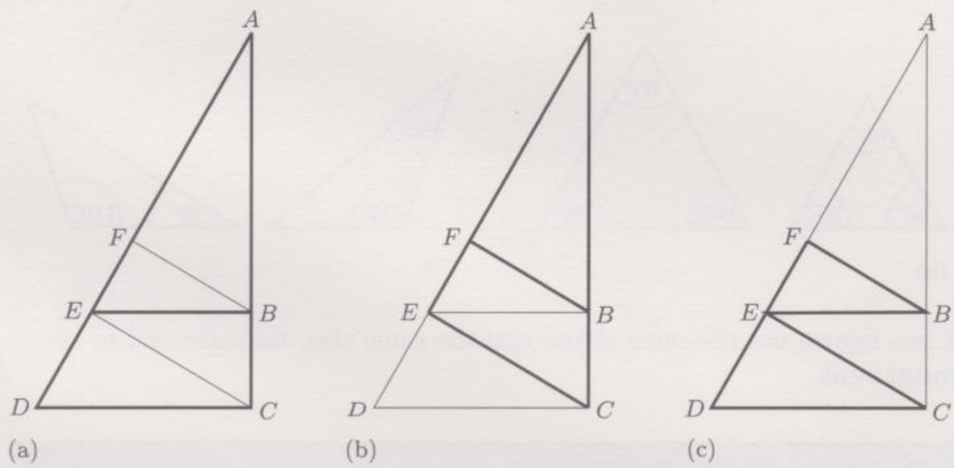
Consider the angles of $\triangle ACD$: $D\widehat{C}A = 90°$ and $C\widehat{D}A = 65°$.

Then

$$\begin{aligned} C\widehat{A}D &= 180° - 90° - 65° \\ &= 25°. \end{aligned}$$

It is easiest to see which triangles are similar if you look at them in pairs.

In each diagram, the two triangles under consideration are emphasized by heavy lines.



In (a), both the triangles that are outlined by heavy lines have the same angle at A, 25°, and both also have a right angle (at C and B, respectively). Therefore the third angle in the two triangles (at D and E) must also be the same. (You can confirm this by noticing that these are corresponding angles.) The size of these angles must be $180° - 25° - 90° = 65°$.

In (b), both triangles have the same angle at A, 25°, and they both have a right angle (at E and F, respectively). Then the third angle in each will be the same size, 65°.

In (c), each triangle has a right angle (at E and F, respectively), and $E\widehat{D}C$ in the larger triangle is the same size as $F\widehat{E}B$ in the smaller triangle (they are corresponding angles). These corresponding angles are each 65°; hence the third angle must again be 25°.

This gives six triangles, each with angles of 25°, 90° and 65°, and so all are similar.

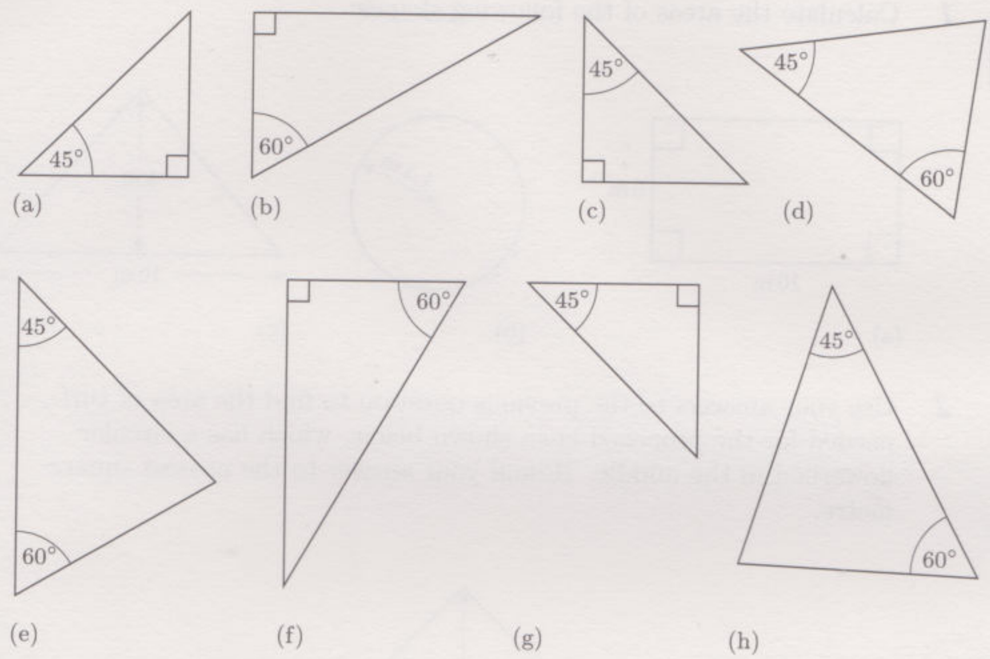
There is a seventh triangle that is also similar to the others, $\triangle BEC$. This has a right angle, and its angle $B\widehat{E}C = E\widehat{C}D$ in $\triangle ECD$ (they are alternate angles), and so is 25°. Its third angle must therefore be 65°.

You have met other examples of similar shapes in the section on scale diagrams in Module 5. Thus the scale plan of a house is similar to the actual layout of the house.

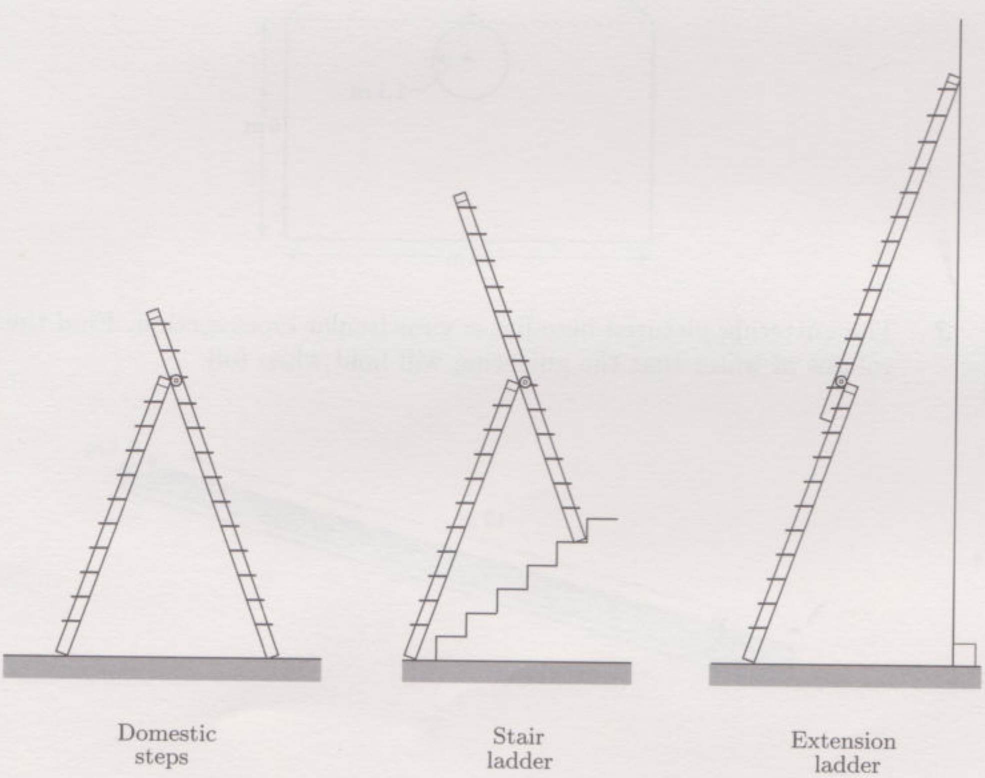
Try some yourself (7.2.5)

Solutions on page 111.

1 Which of these triangles are similar?



2 An aluminium ladder can be used in three different ways:



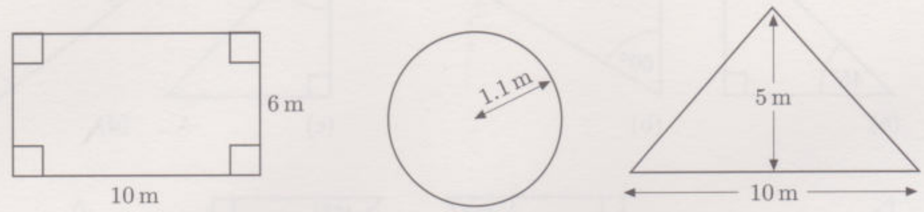
The manufacturer says that in use, each segment of the ladder should make an angle of 20° with the vertical.

For each diagram, add construction lines and labels so as to identify two similar triangles. Are any of the similar triangles also congruent?

7.3 Areas and volumes

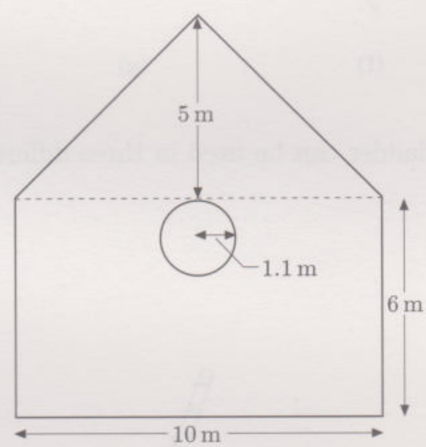
Try these first

1 Calculate the areas of the following shapes:

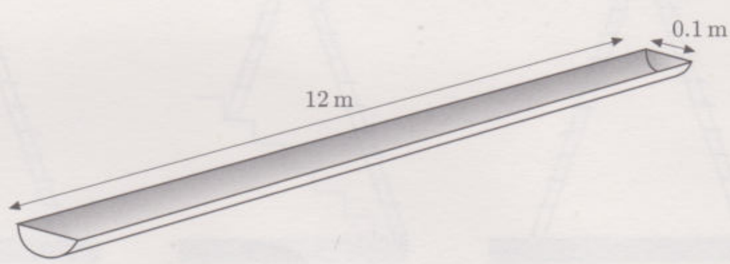


(a) (b) (c)

2 Use your answers to the previous question to find the area of turf needed for the proposed lawn shown below, which has a circular flowerbed in the middle. Round your answer to the nearest square metre.



3 The guttering pictured here has a semicircular cross-section. Find the volume of water that the guttering will hold when full.



Check your answers

1 (a) Area $= 10\,\text{m} \times 6\,\text{m} = 60\,\text{m}^2$.

(b) Area $= \pi(1.1\,\text{m})^2 \simeq 3.80\,\text{m}^2$.

(c) Area $= \frac{1}{2} \times 10\,\text{m} \times 5\,\text{m} = 25\,\text{m}^2$.

Follow-up in Sections 7.3.1 and 7.3.2.

2 Add together the areas of the rectangle and the triangle from Question 1, and subtract the area of the circle to find

$$\begin{aligned}\text{area of turf needed} &= (60 + 25 - 3.80)\,\text{m}^2 \\ &\simeq 81\,\text{m}^2 \text{ (to the nearest square metre).}\end{aligned}$$

Follow-up in Sections 7.3.1 and 7.3.2.

3 The cross-section of the guttering is a semicircle of radius 0.05 m. So

$$\text{area of semicircular cross-section} = \tfrac{1}{2}\pi(0.05\,\text{m})^2 \simeq 0.003\,927\,\text{m}^2.$$

Then

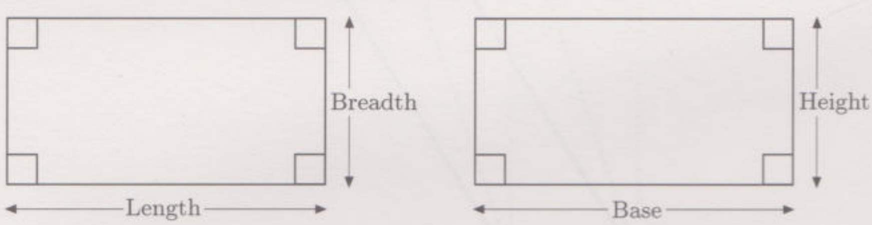
$$\text{volume of guttering} \simeq 12\,\text{m} \times 0.003\,927\,\text{m}^2 \simeq 0.047\,12\,\text{m}^3 \text{ (to 5 d.p.).}$$

Therefore the guttering will hold about $0.047\,\text{m}^3$ of water.

Follow-up in Section 7.3.3.

7.3.1 Areas of quadrilaterals and triangles

The simplest areas to find are those of rectangles. The area of a rectangle is its length multiplied by its breadth. Sometimes the dimensions of a rectangle are referred to as the base and the height, instead of the length and the breadth. The area is then expressed as the base multiplied by the height.



Area of a rectangle = length × breadth = base × height

A square is a special kind of rectangle in which the length is equal to the breadth. Hence its area is the length of one side multiplied by itself, or the length of one side squared.

Area of a square = length of side × length of side = (length of side)^2

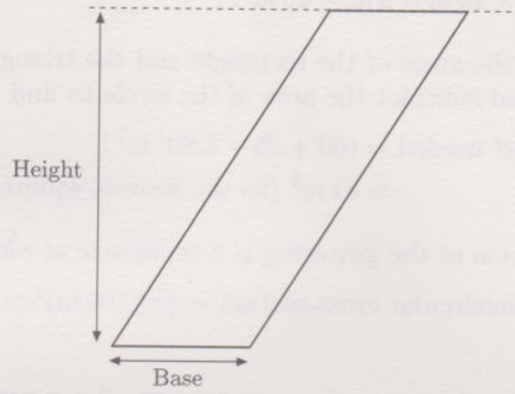
For example, the area of a square mirror with sides 50 cm long is $50\,\text{cm} \times 50\,\text{cm} = 2500\,\text{cm}^2$.

Now consider parallelograms.

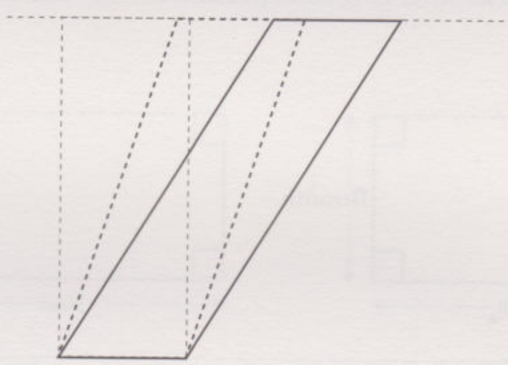
Area of a parallelogram = base × height

The height is *not* the length of the sloping side.

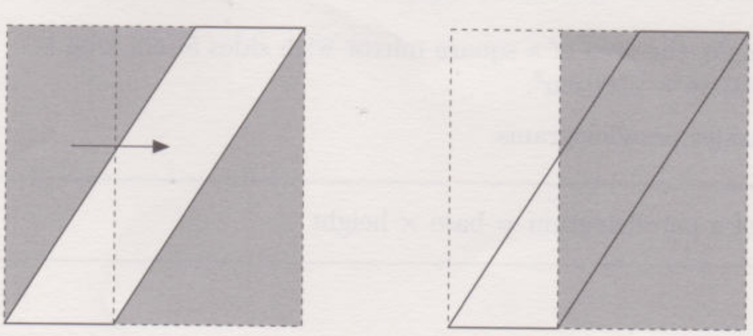
In the formula for the area of a parallelogram, the height is the perpendicular distance from the base to the opposite side. In order to avoid ambiguity it is sometimes called the *perpendicular height* rather than just the height.



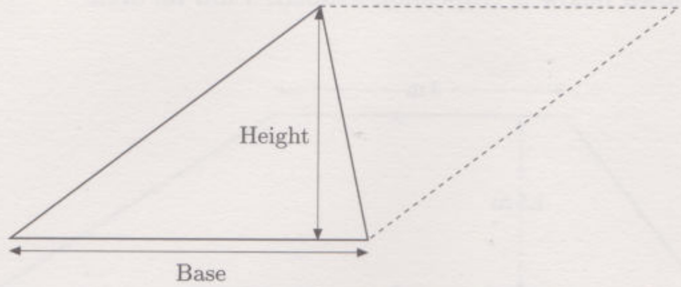
At first sight, the formula for a parallelogram is quite surprising: it is the same formula as that for a rectangle. Imagine the bottom side of the parallelogram is fixed, but the top side slides along a line, as in the diagram below. The top and bottom of the parallelogram remain the same length and the same distance apart, while the other two sides lengthen or shrink. The shape always remains a parallelogram. (Notice that in one position, the parallelogram will become a rectangle—its sides will be at right angles to the base.)



The area of the parallelogram stays the same as the parallelogram shifts: it is equal to the area of the rectangle (which, of course, is given by base × height). This is easy to see by looking at the next diagram. In this, the first figure consists of two identical triangles and a parallelogram. Imagine the left-hand triangle slides to the right: it will fit above the other triangle and leave a rectangle to the left. The second figure shows the same two triangles and the rectangle. Therefore the area of the parallelogram must be the same as the area of the rectangle.



Next think about the areas of triangles. Any triangle can be seen as half of a parallelogram.

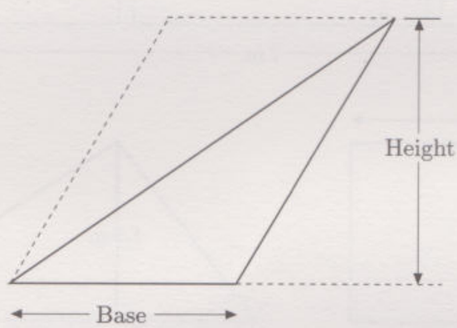


So the area of a triangle is half the area of a parallelogram.

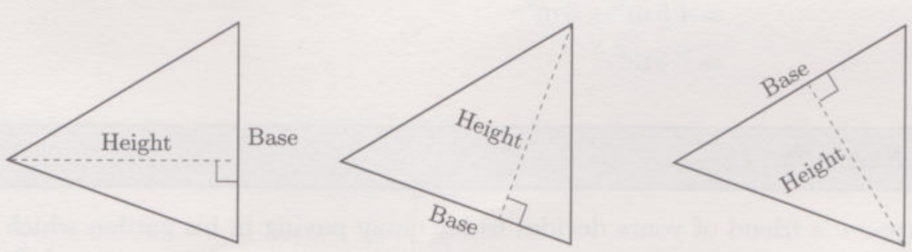
> Area of a triangle = $\frac{1}{2}\times$ base $\times$ height.

Again, the height is the perpendicular height, which is now the distance from the base to the opposite corner, or vertex, of the triangle.

This formula is true for any triangle, because any triangle will be half of a parallelogram even when the perpendicular height lies outside the triangle, as below.



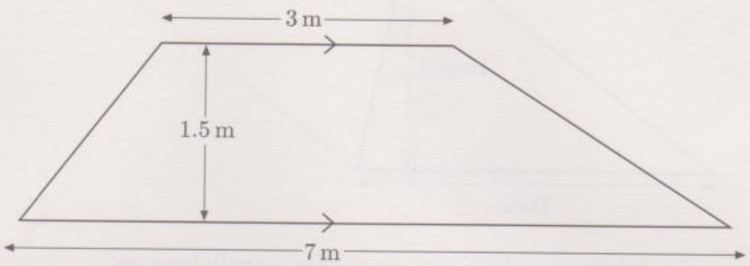
If a triangle does not have a side that is horizontal, it is not clear which side is 'the base'. The beauty of the formula for the area is that it works no matter which side is called 'the base'. Thus the area of the following triangle can be evaluated in three ways.



You can often use what you know about the areas of rectangles and triangles to find the areas of more complex shapes.

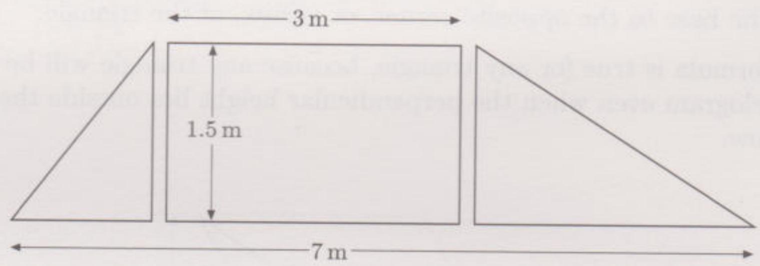
Example 11

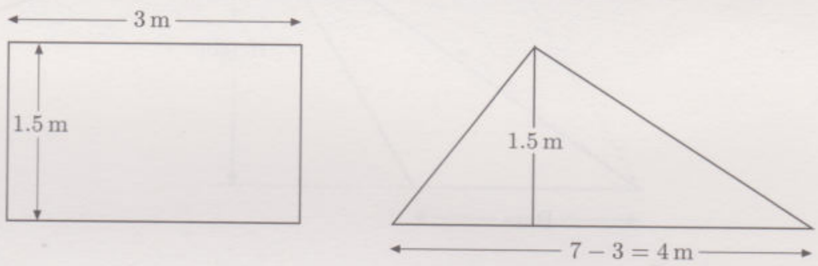
The lawn shown below is trapezium-shaped. Find its area.



Solution

Divide the lawn into three parts—a rectangle and two triangles. Then combine the two triangles into one.





So

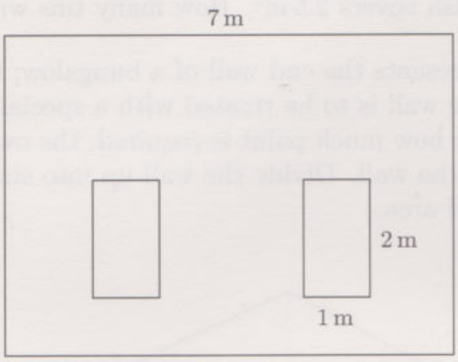
$$\begin{aligned} \text{area of lawn} &= (\text{area of rectangle}) + (\text{area of triangle}) \\ &= (3\,\mathrm{m} \times 1.5\,\mathrm{m}) + (\tfrac{1}{2} \times 4\,\mathrm{m} \times 1.5\,\mathrm{m}) \\ &= 4.5\,\mathrm{m}^2 + 3\,\mathrm{m}^2 \\ &= 7.5\,\mathrm{m}^2. \end{aligned}$$

Example 12

Suppose a friend of yours decides to lay crazy paving in his garden which measures 7 m by 5 m, but he wants to leave two rectangular areas, each 2 m by 1 m, for flowerbeds. If it costs £8.50 to cover an area of 1 m^2 with crazy paving, how much will it cost your friend to pave his garden?

Solution

The first thing to do when tackling a problem like this is to draw a diagram, and to include on it all the information that has been given.



Note that, as the positions of the flowerbeds have not been specified, it does not matter where they are placed.

From the diagram,

area of garden $= 7\,\text{m} \times 5\,\text{m} = 35\,\text{m}^2$,

area of one flowerbed $= 2\,\text{m} \times 1\,\text{m} = 2\,\text{m}^2$.

Therefore,

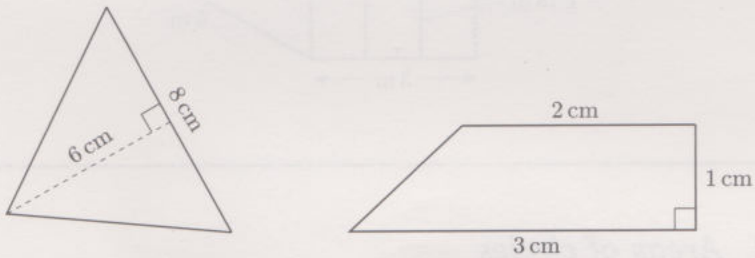
$$\begin{aligned}\text{area of crazy paving} &= \text{area of garden} - (2 \times \text{area of one flowerbed})\\ &= 35\,\text{m}^2 - (2 \times 2\,\text{m}^2) = 35\,\text{m}^2 - 4\,\text{m}^2 = 31\,\text{m}^2.\end{aligned}$$

So your friend will need to buy enough crazy paving to cover $31\,\text{m}^2$. This will cost $31 \times £8.50 = £263.50$.

Try some yourself (7.3.1)

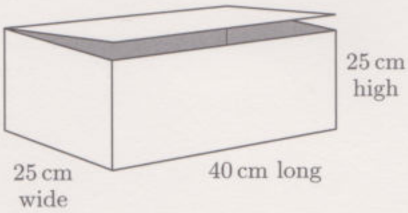
Solutions on page 112.

1 Find the area of each of these shapes.



(a) (b)

2 A girl is decorating a box by glueing wrapping paper on each face. She wants to put paper on the sides, the top and the bottom, and intends to cut out six pieces of paper and stick them on. Assuming no wastage, calculate what area of paper she will need.

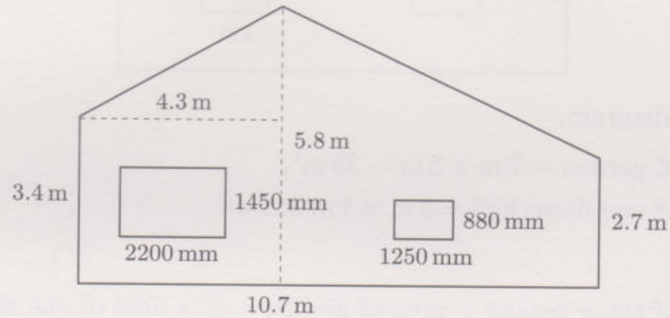


3 A rug measures 3 m by 2 m. It is to be laid on a wooden floor that is 5 m long and 4 m wide. The floorboards not covered by the rug are to be varnished.

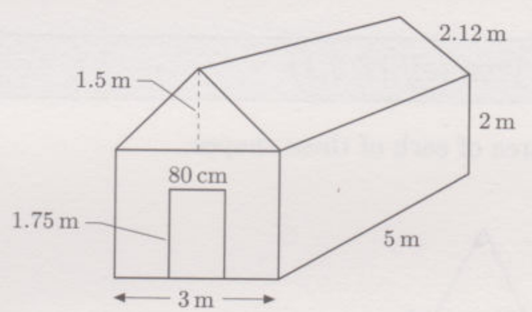
(a) What area of floor will need to be varnished?

(b) A tin of varnish covers $2.5\,\text{m}^2$. How many tins will be required?

4 This diagram represents the end wall of a bungalow; the wall contains two windows. The wall is to be treated with a special protective paint. In order to decide how much paint is required, the owner wants to know the area of the wall. Divide the wall up into simple shapes and then find the total area.



5 The diagram below shows the dimensions of a frame tent. Calculate the amount of canvas needed to make the tent, ignoring the door which is made of different material.



7.3.2 Areas of circles

There are two very famous formulas for circles:

circumference of a circle $= \pi \times$ diameter

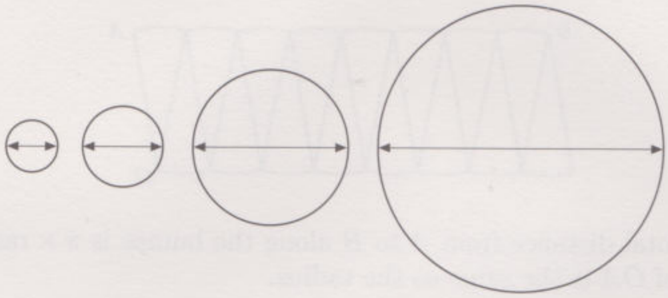
and

area of a circle $= \pi \times (\text{radius})^2$.

Why are these formulas true?

The formula for the circumference of a circle has the following rationale.

It is obvious that as the diameter of a circle increases, so does the circumference:



However, because all circles are the same shape, the relationship between the diameter and the circumference must be the same for every circle. The circumference is always a constant number multiplied by the diameter:

circumference = constant × diameter.

The constant number is given the name 'pi' and is written π. Its value is approximately 3.14. Therefore:

π is the Greek letter for p.

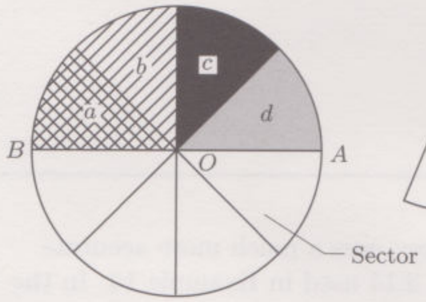
Circumference of a circle = π × diameter

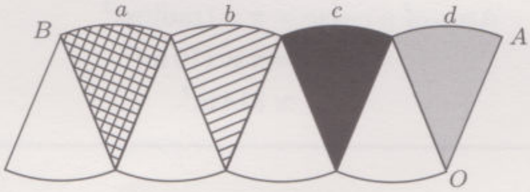
Since the diameter is twice the radius, this formula can be written as

circumference = $\pi \times 2 \times$ radius = $2\pi \times$ radius.

The formula for the area of a circle can also be explained quite simply, as outlined below.

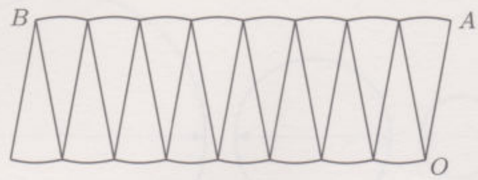
The circle here has been divided into equal 'slices' or **sectors**. The eight sectors can then be cut out and rearranged into the shape shown: this shape has the same area as the circle.





You can see that the total distance from A to B along the 'bumps' is the same as half the circumference of the circle, that is $\frac{1}{2} \times 2\pi \times$ radius = $\pi \times$ radius. Also the length OA is the same as the radius of the circle.

Imagine dividing the circle into more and more sectors and rearranging them as described on the preceding page. For example, dividing the circle into 16 equal sectors gives the following shape, whose area is still the same as that of the circle.



Again the total distance from A to B along the bumps is $\pi \times$ radius, and the length of OA is the same as the radius.

Notice how the rearranged shape is beginning to look more like a rectangle. The more sectors, the straighter AB will become and the more perpendicular OA will be. Eventually it will not be possible to distinguish the rearranged shape from a rectangle. The area of this rectangle will be the same as that of the circle, and its sides will have the lengths $\pi \times$ radius (for AB) and radius (for OA). So the following formula can be deduced:

$$\begin{aligned}\text{area of a circle} &= \text{area of an equivalent rectangle}\\ &= \text{length} \times \text{breadth}\\ &= (\pi \times \text{radius}) \times \text{radius}\\ &= \pi \times (\text{radius})^2.\end{aligned}$$

Area of a circle $= \pi \times (\text{radius})^2$

Example 13

A circular flowerbed is situated in the centre of a traffic roundabout. The radius of the flowerbed is 10 m. Find its area.

Solution

$$\begin{aligned}\text{Area of a circle} &= \pi \times (\text{radius})^2\\ &= \pi \times (10\,\text{m})^2\\ &\simeq 314\,\text{m}^2.\end{aligned}$$

Using $\pi \simeq 3.14$.

Use of the π key is explained in Section 1.7 of Chapter 1 of the *Calculator Book*.

The course calculator has a π key. This key uses a much more accurate value of π than the approximate value of 3.14 used in Example 13. In the exercises that follow you may choose to use your calculator, with its accurate π value, rather than the approximate value 3.14.

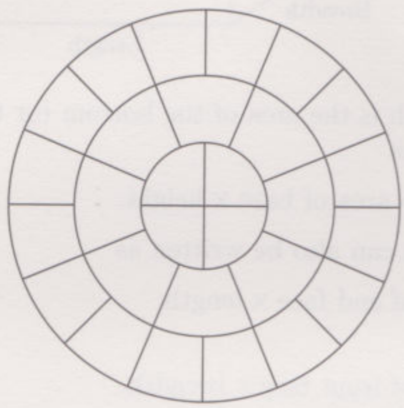
Try some yourself (7.3.2)

Solutions on page 113.

1 Find the area of a circle of (a) radius 8 cm, and (b) radius 15 m.

2 A manufacturer produces a patio kit consisting of 28 paving slabs which are shaped so that they fit together to form rings, as shown. The outside edge of each ring is a circle, and all three circles have the same centre. The slabs are of three sizes, one for each ring of the patio. All of the slabs in a particular ring are identical.

Circles with the same centre are called **concentric circles**.



The radii of the three circles are 0.4 m, 0.8 m and 1.2 m.

Making appropriate assumptions, calculate which of the three types of slab is the heaviest and which the lightest.

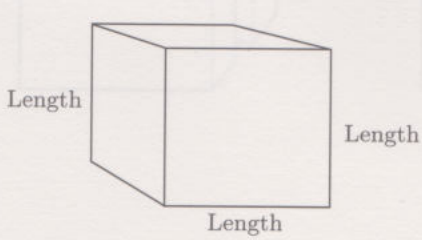
7.3.3 Volumes

What *is* a volume? The word usually refers to the amount of three-dimensional space that an object occupies. It is commonly measured in cubic centimetres (cm^3) or cubic metres (m^3).

A closely related idea is *capacity*; this is used to specify the volume of liquid or gas that a container can actually hold. You might refer to the volume of a brick and the capacity of a jug—but not vice versa. Note that a container with a particular volume will not necessarily have the same amount of capacity. For example, a toilet cistern will have a smaller capacity than its total volume because the overflow pipe makes the volume above the pipe outlet unusable. Some units are used *only* for capacity—examples are litre, gallon and pint; cubic centimetres and cubic metres can be used for either capacity or volume.

One of the simplest solid shapes is a **cube**; it has six identical square faces.

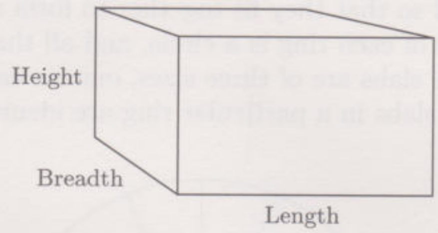
Volume of a cube = length × length × length = $(\text{length})^3$



The technical term for a rectangular box is a **cuboid**.

A shape that resembles a cube is a rectangular box.

Volume of a rectangular box = length × breadth × height



The length × breadth is the area of the bottom (or top) of the box, so an alternative formula is

volume of box = area of base × height.

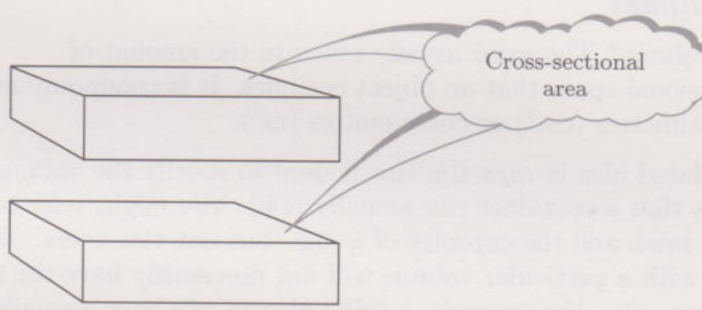
The volume formula can also be written as

volume = area of end face × length

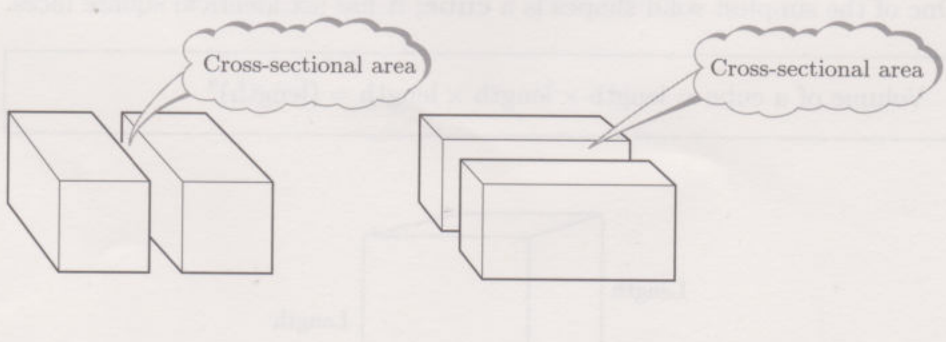
or

volume = area of front face × breadth.

An important idea when calculating volumes of simple shapes is that of a **cross-section**. In the case of the rectangular box considered above, it is possible to slice through the box horizontally so that the sliced area is exactly the same as the area of the base or top; in other words, the areas of the horizontal cross-sections are equal.



Likewise, you could slice through the box vertically in either of two different directions, producing cross-sections that are the same as either the end faces or the front and back faces.

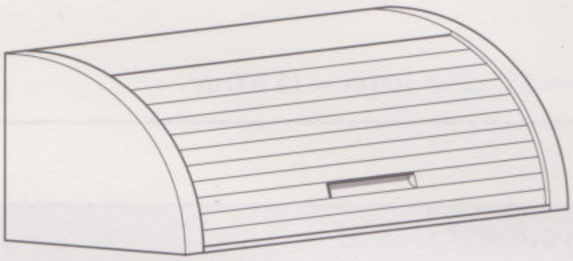


For objects that have a *constant cross-sectional area*, there is a very useful formula for the volume.

> Volume =
> cross-sectional area × length at right angles to the cross-section

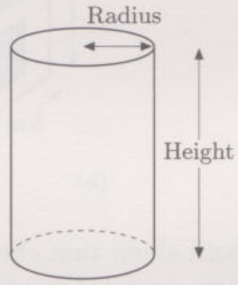
Notice that this fits with the formula for the volume of a rectangular box.

Bear in mind that many objects can only be sliced in one direction to produce a constant cross-sectional area. This bread bin is one example.



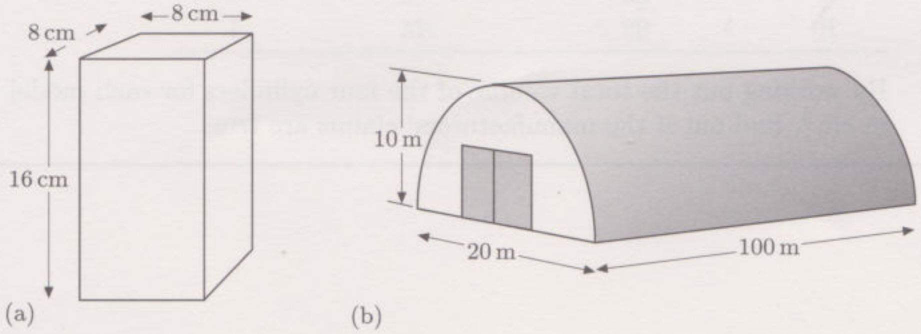
Another example is a cylinder. The formula at the top of the page can be used to find the volume of a cylinder because a cylinder has a constant cross-sectional area if it is sliced parallel to the circular face—the cross-sectional area is the area of the circle that forms the base of the cylinder, that is $\pi \times (\text{radius})^2$. The following formula can then be deduced.

> Volume of cylinder = $\pi \times (\text{radius})^2 \times$ height



Example 14

Find the volumes of these objects.



Solution

(a) For this object,

$$\text{cross-sectional area} = 8\,\text{cm} \times 8\,\text{cm} = 64\,\text{cm}^2,$$

therefore

$$\text{volume} = 16\,\text{cm} \times 64\,\text{cm}^2 = 1024\,\text{cm}^3.$$

(b) For this object,

$$\text{cross-sectional area} = \text{area of semicircle} = \tfrac{1}{2} \times \pi \times (\text{radius})^2$$
$$= \tfrac{1}{2} \times \pi \times (10\,\text{m})^2 \simeq 157\,\text{m}^2,$$

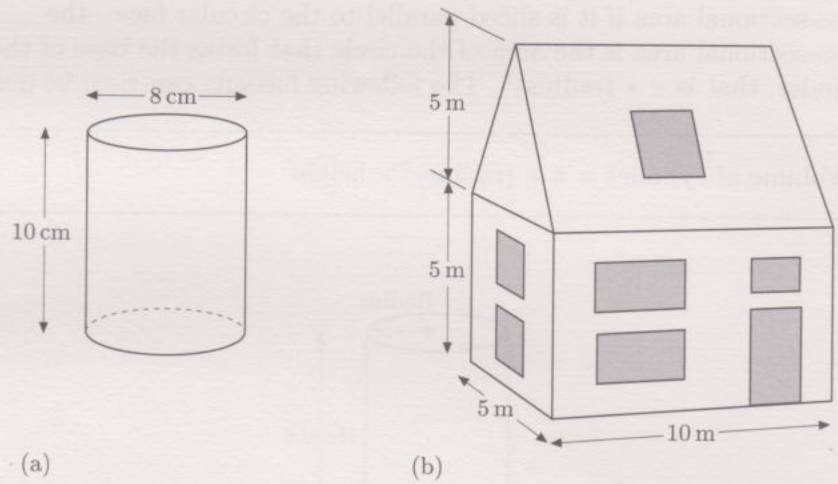
therefore

$$\text{volume} = 157\,\text{m}^2 \times 100\,\text{m} = 15\,700\,\text{m}^3.$$

Solutions on page 114.

Try some yourself (7.3.3)

1 Find the volumes of these objects.



2 Two car manufacturers both claim that their models have an engine capacity of 2 litres. The two models have four-cylinder, four-stroke engines.

The table below shows the details of the four cylinders.

Car model	Cylinder diameter (bore)/mm	Cylinder height (stroke)/mm	Number of cylinders
A	86	86	4
B	92	75	4

1 litre $= 1000\,\text{cm}^3$.

By working out the total volume of the four cylinders for each model in cm^3, find out if the manufacturers' claims are true.

7.3.4 Scaling areas and volumes

In Module 5 you saw how a scale is used on plans of houses and other structures. The scale makes it possible to take a length on the plan and calculate the corresponding length in reality. The scale can also be used to convert between areas on the plan and real areas. Moreover, if a three-dimensional scale *model* is made, it is possible to use the scale to convert between volumes in the model and the real volumes.

Example 15

Dudley's and June's hobby is constructing doll's houses. They decide to make a model of their own house, using a scale in which 1 cm on the model represents 20 cm on their real house.

They are making the curtains for the model. The window in the real dining room measures 240 cm by 120 cm. What is the area of the real window and the area of the window in the model? How many times greater is the real area?

Solution

The real window has an area of $240\,\text{cm} \times 120\,\text{cm} = 28\,800\,\text{cm}^2$. It might be easier to think of this in square metres, that is

$$\begin{aligned}
&(240\,\text{cm} \div 100) \times (120\,\text{cm} \div 100)\\
&= 2.4\,\text{m} \times 1.2\,\text{m}\\
&= 2.88\,\text{m}^2.
\end{aligned}$$

To find the dimensions of the window in the model, divide the real lengths by 20 as the scale is 1 cm to 20 cm:

$$\begin{aligned}
\text{length} &= 240\,\text{cm} \div 20 = 12\,\text{cm},\\
\text{width} &= 120\,\text{cm} \div 20 = 6\,\text{cm},\\
\text{area} &= 12\,\text{cm} \times 6\,\text{cm} = 72\,\text{cm}^2,
\end{aligned}$$

or in square metres,

$$\begin{aligned}
\text{length} &= 2.4\,\text{m} \div 20 = 0.12\,\text{m},\\
\text{width} &= 1.2\,\text{m} \div 20 = 0.06\,\text{m},\\
\text{area} &= 0.12\,\text{m} \times 0.06\,\text{m} = 0.0072\,\text{m}^2.
\end{aligned}$$

Now find the number of times that the area of the real window exceeds the area of the window in the model:

working in square centimetres $\quad 28800 \div 72 = 400,$

or

working in square metres $\quad 2.88 \div 0.0072 = 400.$

As the real lengths are 20 times greater than those on the model, the areas are $20^2 (= 400)$ times greater.

This example has demonstrated a general result:

> To scale areas, multiply or divide by the scale squared.

Example 16

Dudley and June have a cold-water tank in their loft, which has a capacity of 250 litres. If they make a scale model of the tank, what will its capacity be?

Solution

Just as areas must be multiplied or divided by the scale squared, so volumes (and capacities) must be multiplied or divided by the cube of the scale. Here the capacity of the real tank must be divided by $20^3 (= 8000)$. Therefore

$$\text{capacity of model tank} = 250 \text{ litres} \div 8000 = 0.03125 \text{ litres}.$$

As there are $1000\,\text{cm}^3$ in one litre,

$$\text{capacity} = 0.03125 \times 1000\,\text{cm}^3 = 31.25\,\text{cm}^3.$$

A check on this value can be made by considering the volume of the real water tank. If it is assumed that the full tank holds exactly 250 litres, the volume of the tank would be at least $250 \times 1000\,\text{cm}^3 = 250\,000\,\text{cm}^3$.

The question does not give the dimensions of the real tank, but to produce this volume, the dimensions might perhaps be 50 cm by 50 cm by 100 cm. (Note that $50 \times 50 \times 100 = 250\,000$.) The dimensions of the model of such a tank would be

$$(50\,\text{cm} \div 20) \times (50\,\text{cm} \div 20) \times (100\,\text{cm} \div 20)$$
$$= 2.5\,\text{cm} \times 2.5\,\text{cm} \times 5\,\text{cm}.$$

So

$$\text{volume of model tank} = 31.25\,\text{cm}^3.$$

This example illustrates a general result:

> To scale volumes, multiply or divide by the scale cubed.

Solutions on page 114.

Try some yourself (7.3.4)

1 Calculate the area of a carpet in a model house if the real carpet has an area of $22\,\text{m}^2$. On the scale used, 1 cm represents 0.25 m.

2 A model steam engine that runs in a park is built to a scale such that 1 cm represents 0.2 m. On the model there is space in the tender for $\frac{1}{200}\,\text{m}^3$ of coal. What volume of coal could be carried in the real engine's tender?

Outcomes

Now that you have studied Module 7 you should be able to:

- ◇ understand what is meant by the terms: quadrilateral, rectangle, square, parallelogram, trapezium, triangle, right-angled triangle, isosceles triangle, equilateral triangle and scalene triangle;
- ◇ understand what is meant by the terms: circle, circumference, radius, diameter, semicircle and sector;
- ◇ determine the lines of symmetry, the centre of rotation and the order of rotational symmetry for simple shapes;
- ◇ understand what is meant by an angle and a vertex, and determine whether an angle is acute, right, obtuse, or reflex;
- ◇ measure angles in degrees, and use appropriate notation for angles;
- ◇ understand what is meant by parallel lines;
- ◇ recognize vertically opposite, corresponding and alternate angles, and use facts about these to determine unknown angles;
- ◇ state the sum of the angles at a point and on a line, and also the sum of the angles of a triangle and of a quadrilateral, and use these facts to determine unknown angles;
- ◇ recognize similar and congruent triangles;
- ◇ find the areas of triangles and quadrilaterals of all types;
- ◇ find the area and circumference of any circle;
- ◇ find the areas of shapes based on triangles, quadrilaterals and circles;
- ◇ find the volumes of boxes, cylinders and other objects with constant cross-sectional areas.

Solutions

Section 5.1

1

The window in the diagram is 1.1 squares wide, so in reality it is 1.1 m wide.

The wash basin in the diagram is $\frac{3}{5}$ of a square deep by $\frac{9}{10}$ of a square wide, so in reality it is 0.6 m by 0.9 m.

2

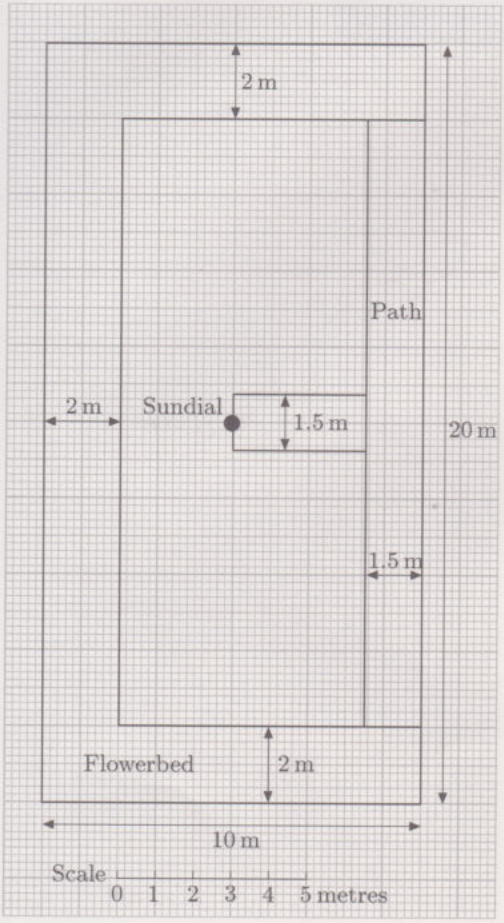
On the diagram, 5 cm represent 1 m.
As 10 cm are 2×5 cm, they represent 2 m.
As 20 cm are 4×5 cm, they represent 4 m.
As 1 cm is $\frac{1}{5} \times 5$ cm, it represents $\frac{1}{5}$ m or 0.2 m.

3

On the map, 1 km is represented by 2 cm.
Thus
10 km are represented by $10 \times 2\,\text{cm} = 20\,\text{cm}$;
5 km are represented by $5 \times 2\,\text{cm} = 10\,\text{cm}$;
0.5 km is represented by $0.5 \times 2\,\text{cm} = 1\,\text{cm}$.

4

Your scale plan should look something like this:



Section 5.2.1

1

(a) 15 minutes.

(b) The temperature drops from 90°C to 36°C in the first 20 minutes, a total of 54 degrees. In the second 20 minutes, the temperature drops from 36°C to 25°C, a total of 11 degrees.

(c) The tea cools off quickly at the beginning when it is hot, then cools more slowly when it is cooler.

2

(a) The 16:21 from Doncaster should get to Wellingborough at 19:04 (changing at Sheffield). The journey time would be 2 hours 43 minutes.

(b) To arrive in Bedford by 7 p.m. (19:00), it would be necessary to catch the 16:23 train from Derby, arriving at Bedford at 17:54.

3

(a) The Potters would be charged £289 each, plus a supplement of £4.45 per person per night. Their holiday would cost

$$(2 \times £289) + (2 \times 7 \times £4.45) = £640.30.$$

(b) One-bedroom self-catering accommodation would cost them £259 each, plus a supplement of £7.80 per person per night:

$$(2 \times £259) + (2 \times 7 \times £7.80) = £627.20.$$

This would result in a saving of

$$£640.30 - £627.20 = £13.10.$$

4

(a) The total number of households surveyed in 1971 was 18.6 million.

(b) The table shows that 16% of the households surveyed in 1998 consisted of three people. The actual number of such households was 16% of 23.6 million, which is 3.776 million.

(c) The percentage total for the 1981 column is

$$22 + 32 + 17 + 18 + 7 + 4 = 100\%.$$

5

(a) The percentage of dependent children in single-parent families is

$$19 + 2 = 21\%.$$

(b) The percentage of dependent children in families comprising couples with two or more children is

$$37 + 25 = 62\%.$$

6

(a) Find '25–34' in the column headings; then look down this column to the 'Over 20' row. This gives 29%.

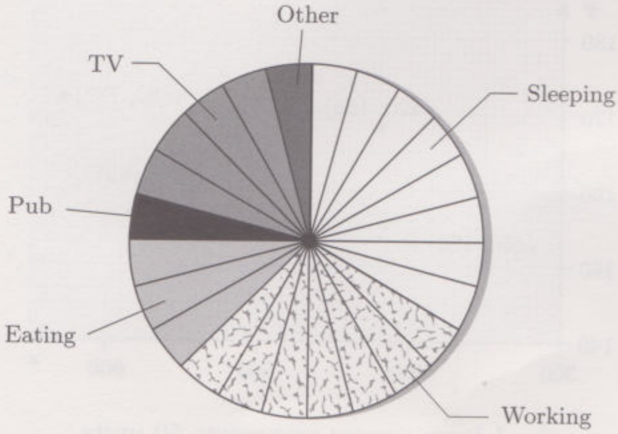
(b) The '60 or over' column shows that 60% of men aged 60 or over do not smoke, and that the total number of men aged 60 or over in the sample is 2150. So 60% of 2150 = 1290 of the men aged 60 or over are non-smokers.

(c) Suppose a 'heavy smoker' is defined as one who smokes more than 20 cigarettes per day. Then, less than 20% of men in the 16–24 age group and the 60 and over group are heavy smokers. In the 25–34, 35–49 and 50–59 age groups, well over 20% are heavy smokers. So these are the age groups with the higher percentages of heavy smokers, with the age range 25–34 having marginally the highest.

Section 5.2.2

1

One way of shading and labelling the chart is shown below.



2

The largest proportion of people (about 33%) live in semi-detached houses. About 30% live in terraced houses, and about 15% live in detached houses. Over 10% live in flats or maisonettes, and about 10% live in some other type of accommodation.

3

(a) Detached houses.

(b) Terraced houses.

(c) The pie chart labelled 'All households', shows that semi-detached houses are the most common type of accommodation, although terraced houses are almost as common.

(d) About 30% of employers/managers live in detached houses compared with only about 10% of skilled manual workers. Marginally more skilled manual workers than employers/managers live in semi-detached houses, though the proportions in this category (about 30%) are very similar. More skilled manual workers (about 30%) than employers/managers (about 15%) live in terraced houses. About the same proportion of each (about 25%) live in either flats/maisonettes or 'other' accommodation.

Section 5.2.3

1

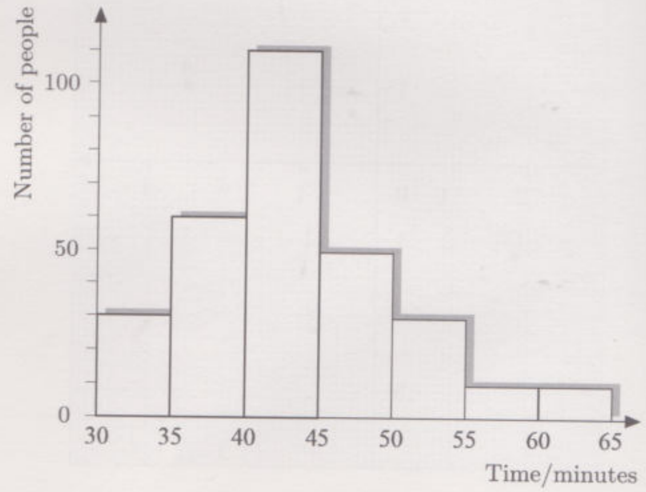
(a) About 12 000.

(b) About 21 000.

(c) Generally the sales figures are increasing, although there was a slight fall in 1994.

2

(a)



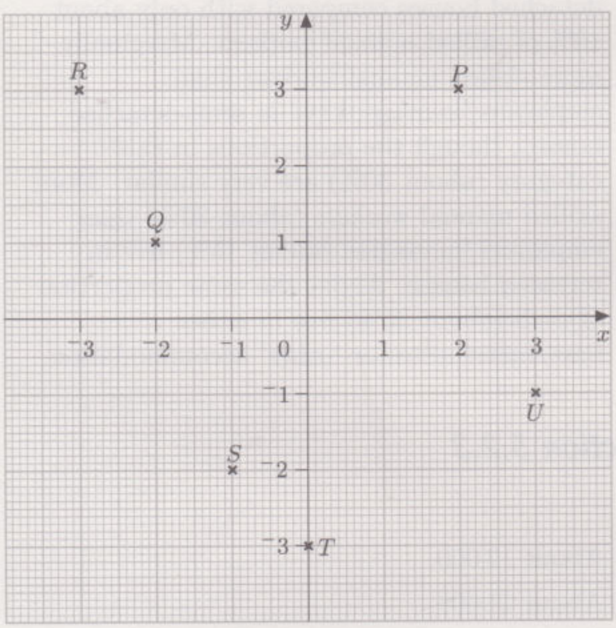
(b) The width of each interval represents 5 minutes of journey time.

Section 5.3.1

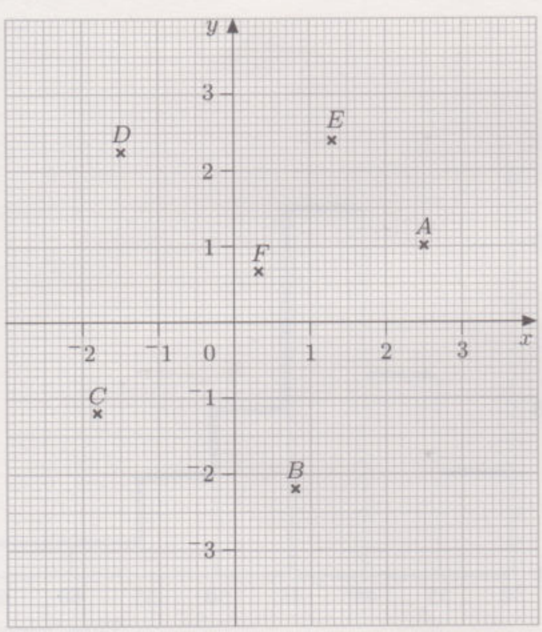
1

The coordinates of A are $(2, 3)$.
The coordinates of B are $(^-1, 2)$.
The coordinates of C are $(^-2, ^-1)$.
The coordinates of D are $(1, ^-2)$.
The coordinates of E are $(^-2, 0)$.

2



3



Note that where a point does not plot exactly onto one of the grid lines, you have to estimate its correct position.

Section 5.3.2

1

(a) $(9.5, 10)$. At 9.30 a.m. the temperature was 10°C.

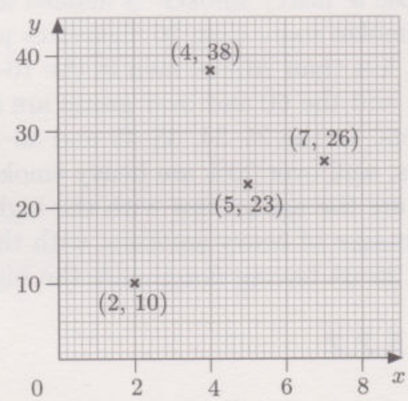
(b) $(900, 1.4)$. The cost of 900 grams (0.9 kg) of washing powder is £1.40.

(c) $(24, ^-60)$. On the 24th of the month there was $^-$£60 in the bank (that is, the account was overdrawn by £60).

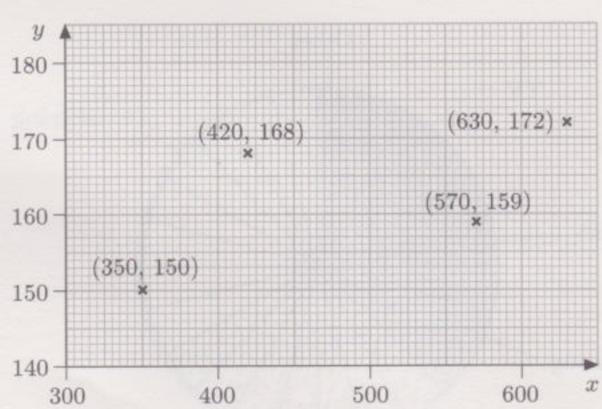
2

The scales you chose will have depended on the size of your graph paper: however, you should have chosen scales that are easy to use for plotting the given points. Your graphs should be something like those below.

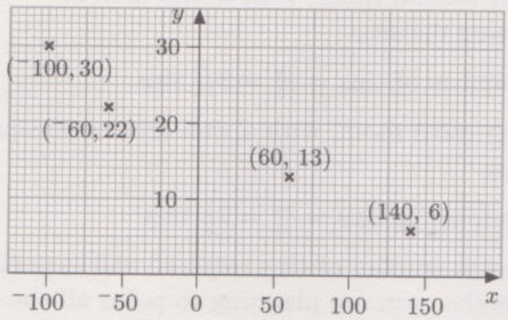
(a) x-axis: 1 large square represents 2 units
y-axis: 1 large square represents 10 units.



(b) x-axis: 1 large square represents 50 units
y-axis: 1 large square represents 10 units.



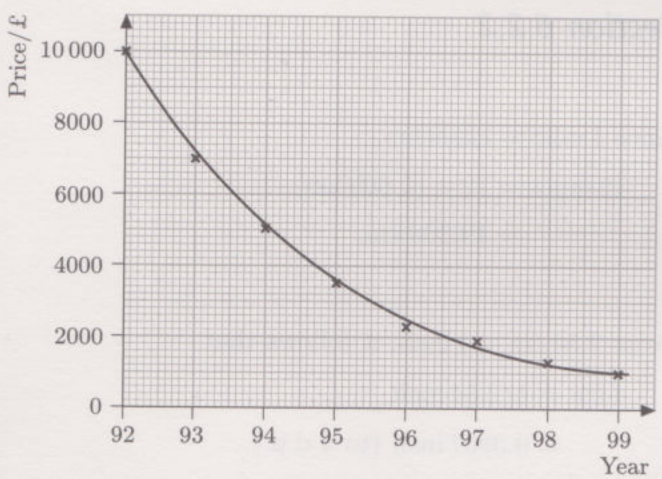
(c) x-axis: 1 large square represents 50 units
y-axis: 1 large square represents 10 units.



3

The cost of coffee increased over the period from £5 per kilogram to over £10 per kilogram. It remained more or less steady between February and April, rose significantly during the month of April, then increased slowly but steadily until July.

4



Section 6.1.1

1

The versions given below are not the only possibilities. You may have used other words, but check that you have written in whole sentences with full stops at the end.

(a) Since

$$2.3+3.7=6 \quad \text{and} \quad 14.8-5.6=9.2,$$

it follows that

$$\frac{2.3+3.7}{14.8-5.6}=\frac{6}{9.2}$$
$$=0.65 \text{ (to 2 d.p.)}.$$

(b) Since

$$(3.2)^2=10.24 \quad \text{and} \quad (8.5)^2=72.25,$$

it follows that

$$(3.2)^2+(8.5)^2=10.24+72.25$$
$$=82.49.$$

2

Since

$$2.3+3.7=6 \quad \text{and} \quad 14.8-5.6=9.2,$$

it follows that

$$\frac{2.3+3.7}{14.8-5.6}=\frac{6}{9.2}=0.652\,173\,913. \qquad (1)$$

Now

$$(3.2)^2=10.24 \quad \text{and} \quad (8.5)^2=72.25.$$

Hence

$$(3.2)^2+(8.5)^2=10.24+72.25=82.49. \qquad (2)$$

So, from (1) and (2),

$$\frac{2.3+3.7}{14.8-5.6}+(3.2)^2+(8.5)^2=0.652\,173\,193+82.49$$
$$=83.14 \text{ (to 2 d.p.)}.$$

Section 6.1.2

1

There are many different correct answers to this question. However, make sure that the meanings you have given are equivalent to the ones below.

Word	Mathematical meaning	Example of use
Decimal	A number expressed in terms of tenths, hundredths, etc.	A quarter expressed as a decimal is 0.25.
Fraction	One whole number over another.	0.75 expressed as a fraction is $\frac{3}{4}$.
Positive	Greater than zero.	2 is a positive number.
Negative	Less than zero.	$^-2$ is a negative number.

Section 6.1.3

1

(a) Add 5 and 8, and then divide the result by the difference between 4 and 2. (This gives $6\frac{1}{2}$.)

(b) Add 5 to the division 8/4, and then subtract 2. (This gives 5.)

(c) Multiply the sum of 4 and 5 by the difference between 5 and 2. (This gives 27.)

(d) Multiply 9 by the square root of 4. (This gives 18.)

(e) Multiply 5 by 6, and then divide by 2. (This gives 15.)

(f) The cube of the square root of 25. (That is, 125.)

(g) The reciprocal of 12 minus 9. (That is, $\frac{1}{3}$.)

2

(a) The mass is greater than or equal to 10 kg.

(b) The time is less than 2.4 million hours.

(c) Two-thirds (or 2 divided by 3) is not equal to 0.67.

Section 6.2.1

1

The formula is

$$\text{cost of tomatoes} = (\text{price per kilogram}) \times (\text{number of kilograms}).$$

The price per kilogram is 75 pence, and the number of kilograms is 1.45. So the formula gives

$$\begin{aligned}\text{cost of tomatoes} &= (75\text{ pence}) \times 1.45\\ &= 108.75\text{ pence.}\end{aligned}$$

Rounded to the nearest penny this is 109 pence, or £1.09.

2

The formula is

$$\text{distance travelled} = (\text{average speed}) \times (\text{time taken}).$$

So, after 1.5 hours,

$$\text{distance travelled} = (60\text{ km per hour}) \times (1.5\text{ hours}) = 90\text{ km}.$$

As 2 hours 40 minutes is $2\frac{2}{3}$ hours, or $\frac{8}{3}$ hours, the formula gives

$$\text{distance travelled} = (60\text{ km per hour}) \times (\tfrac{8}{3}\text{ hours}) = 160\text{ km}.$$

For three and a half hours (3.5 hours), the formula gives

$$\text{distance travelled} = (60\text{ km per hour}) \times (3.5\text{ hours}) = 210\text{ km}.$$

3

For the first room, with a wall area of 56 square metres,

$$\begin{aligned}\text{number of tins} &= \frac{\text{area of wall}}{\text{area covered by one tin}}\\ &= \tfrac{56}{15} = 3\tfrac{11}{15}\text{ tins.}\end{aligned}$$

For the second room, with a wall area of 38 square metres,

$$\text{number of tins} = \tfrac{38}{15} = 2\tfrac{8}{15}\text{ tins.}$$

For the third room, with a wall area of 40 square metres,

$$\text{number of tins} = \tfrac{40}{15} = 2\tfrac{2}{3}\text{ tins.}$$

The total number of tins required will depend on whether you are planning to paint all the rooms in the same colour or not. If you are, the number of tins required will be

$$\tfrac{56}{15} + \tfrac{38}{15} + \tfrac{40}{15} = \tfrac{134}{15} = 8\tfrac{14}{15},$$

so you will need to buy 9 tins. If all the rooms are to be different colours, you will need to buy 4 tins for the first room, 3 for the second and 3 for the third—a total of 10 tins.

Section 6.2.2

1

Since 1 mile = 1.609 km,

$$\begin{aligned}12\text{ miles} &= 12 \times (1.609\text{ km})\\ &= 19.308\text{ km}.\end{aligned}$$

2

From 2.54 cm = 1 inch, it follows that

$$\begin{aligned}1\text{ cm} &= 1/2.54\text{ inch}\\ &= 0.3937\text{ inch (to 4 d.p.).}\end{aligned}$$

From 453.6 g = 1 lb, it follows that

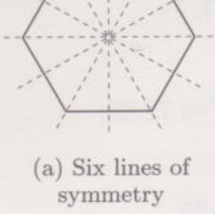
$$\begin{aligned}1\text{ g} &= 1/453.6\text{ lb}\\ &= 0.0022\text{ lb (to 4 d.p.).}\end{aligned}$$

Since 1 kg is 1000 g,

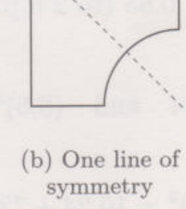
$$1\text{ kg} = 2.2\text{ lb (to 1 d.p.).}$$

Section 7.1.2

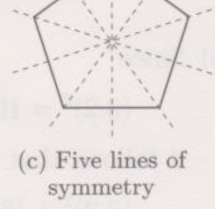
1



(a) Six lines of symmetry



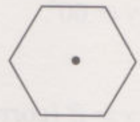
(b) One line of symmetry



(c) Five lines of symmetry

2

(a) Order 2



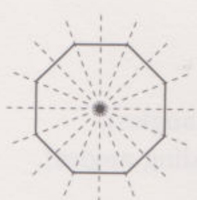
(b) Order 6

(c) Order 5

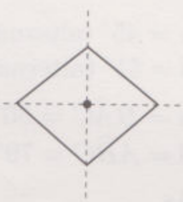
(d) Order 1

Notice that (d) has no rotational symmetry and no centre of rotation.

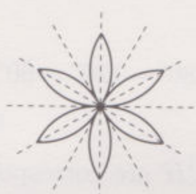
3



(a) Eight lines of symmetry
Order 8



(b) Two lines of symmetry
Order 2



(c) Six lines of symmetry
Order 6

Section 7.2.1

1

Every hour the minute hand turns through 360°. It will have made five such revolutions in five hours. This amounts to 1800°.

The hour hand turns through 30° every hour ($\frac{1}{12}$ of 360°). In five hours it will turn through $5 \times 30° = 150°$.

2

(a) $C\widehat{A}B$ or $\angle CAB$.

(b) $B\widehat{C}A$ or $\angle BCA$.

(c) γ or $\angle DAC$.

(d) δ or $A\widehat{C}D$.

3

(a) 38°: acute.

(b) 120°: obtuse.

(c) 90°: right angle.

(d) 45°: acute.

(e) 155°: obtuse.

4

(a) Red party: 120°.
Blue party: 95°.
Yellow party: 95°.
Green party: 50°.

(b) $120° + 95° + 95° + 50° = 360°$.

(c) Since 360° represents 100%, 1° will represent $\dfrac{1}{360} \times 100\%$ or $\dfrac{100}{360}\%$.

So the Red party polled

$$120 \times \frac{100}{360} \simeq 33\%.$$

The Blue party and the Yellow party polled

$$95 \times \frac{100}{360} \simeq 26\%.$$

The Green party polled

$$50 \times \frac{100}{360} \simeq 14\%.$$

Section 7.2.2

1

(a) Each of the four angles is $360° \div 4 = 90°$.

(b) The two upper angles are both $180° \div 2 = 90°$, and the lower angle is 180°.

(c) Each of the six angles is $360° \div 6 = 60°$.

(d) Each of the twenty angles is $360° \div 20 = 18°$.

(e) The acute angle between the hands is $360° \div 12 = 30°$; the reflex angle is $360° - 30° = 330°$.

(f) Each of the three angles is $360° \div 3 = 120°$.

2

(a) $\gamma = 180° - 90° - 60° = 30°$.

(b) $\delta = 360° - 130° - 60° - 60° = 110°$.

3

(a) 130°, 50°, 130°.

(b) 120°, 60°, 120°.

(c) 90°, 90°, 90°.

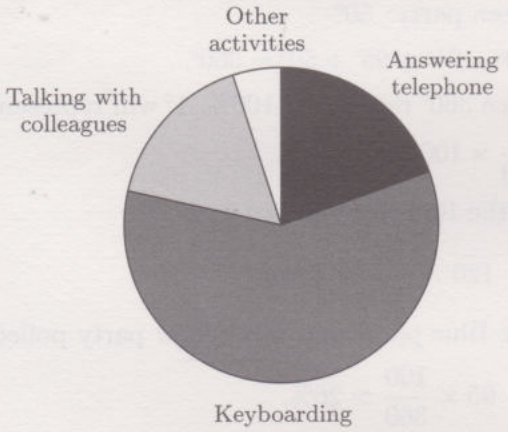
4

Since one hour will be represented by 360° on the pie chart, 1 minute will be represented by $360° \div 60 = 6°$.

So the required angles on the chart are:

Keyboarding	$35 \times 6° = 210°$
Answering telephone	$12 \times 6° = 72°$
Talking with colleagues	$10 \times 6° = 60°$
Other activities	$3 \times 6° = 18°$.

Check: $210° + 72° + 60° + 18° = 360°$.



Section 7.2.3

1

(a) Now

$$\beta + 60° = 180°,$$

so

$$\beta = 120°.$$

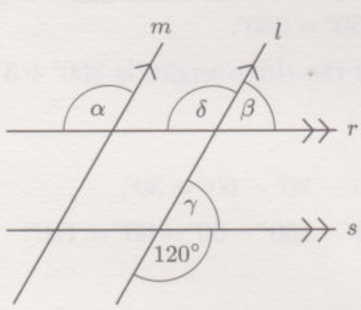
But

$$\alpha = \beta \text{ (corresponding angles)},$$

so

$$\alpha = 120°.$$

(b)



There are many ways of finding the sizes of these angles. This is only one of them:

As

$$\gamma + 120° = 180°,$$

it follows that

$$\gamma = 60°.$$

But

$$\gamma = \beta \text{ (corresponding angles)},$$

so

$$\beta = 60°.$$

Similarly,

$$\delta + \beta = 180°,$$

so

$$\delta = 120°.$$

But

$$\alpha = \delta \text{ (corresponding angles)},$$

so

$$\alpha = 120°.$$

(c) $\alpha = 45°$ (alternate angles).
$\beta = 55°$ (alternate angles).

(d) $\alpha = B\widehat{A}C = 30°$ (alternate angles).
$\beta = A\widehat{B}C = 70°$ (corresponding angles).

(e) As

$$G\widehat{C}B = 40° + 80° = 120°,$$

it follows that

$$B\widehat{C}H = 180° - 120° = 60° \text{ (angles on a straight line)}.$$

But β and $B\widehat{C}H$ are corresponding angles, so

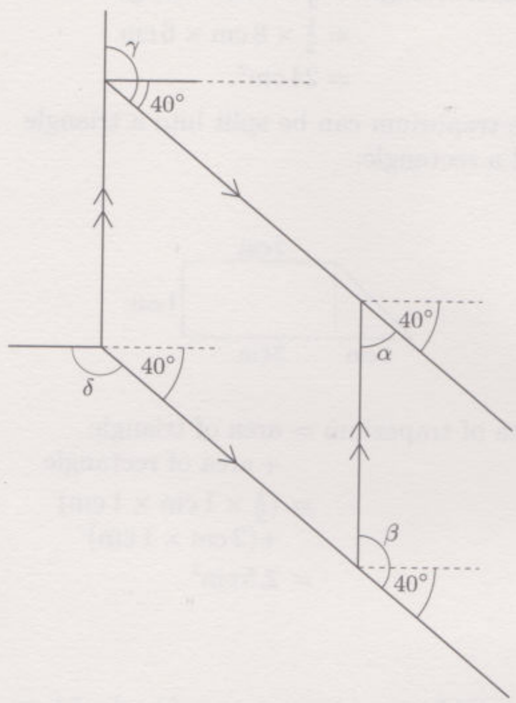
$$\beta = 60°.$$

Whereas α and $B\widehat{C}H$ are alternate angles, so

$$\alpha = 60°.$$

2

It is a good idea to sketch a diagram, adding some horizontal lines where necessary.



Assume the lines marked are pairs of parallel lines. Then, since the handrail makes an angle of 40° with the horizontal,

$\alpha = 90° - 40° = 50°,$
$\beta = 90° + 40° = 130°,$
$\gamma = 90° + 40° = 130°,$
$\delta = 180° - 40° = 140°.$

3

$a = 55°$ (angles equal before and after ball impacts),
$b = 180° - 55° - 55°$ (angles on a straight line) $= 70°$,
$c = 55°$ (alternate angles),
$d = 55°$ (angles equal before and after ball impacts),
$e = 55°$ (alternate angles),
$f = 90° - 55°$ (angles at a right angle) $= 35°$.

Section 7.2.4

1

(a) In $\triangle ABC$,

$$\alpha + 130° + 30° = 180°,$$

therefore

$$\alpha = 180° - 130° - 30° = 20°.$$

(b) In $\triangle FGH$,

$$\beta + 90° + 25° = 180°,$$

therefore

$$\beta = 180° - 90° - 25° = 65°.$$

As EFG is a straight line,

$$\theta + \beta = 180°.$$

So

$$\theta = 180° - 65° = 115°.$$

In $\triangle EFH$,

$$\alpha = 180° - 50° - \theta = 15°.$$

2

(a) As this is an isosceles triangle, $\alpha = \beta$.

So

$$60° + 2\alpha = 180°.$$

Therefore

$$2\alpha = 120° \text{ and } \alpha = 60°.$$

(b) As this is an isosceles triangle, $\gamma = \delta$.

So

$$90° + 2\gamma = 180°.$$

Therefore

$$2\gamma = 90° \text{ and } \gamma = 45°.$$

Section 7.2.5

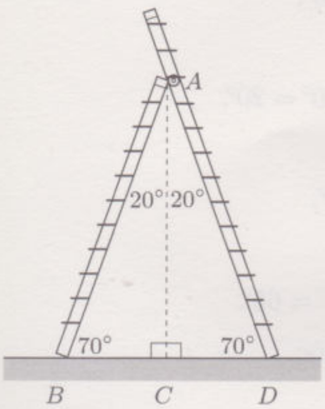
1

Triangles a, c and g are similar since they have angles of 90°, 45° (and hence another angle of 45°).

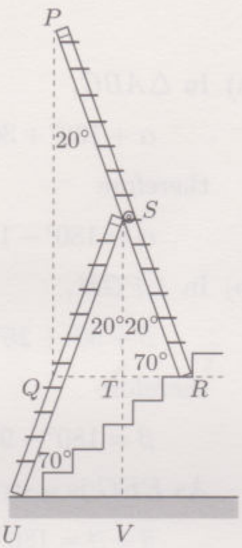
Triangles b and f are similar since they have angles of 90°, 60° (and hence another angle of 30°).

Triangles d, e and h are similar since they have angles of 45° and 60° (and hence another angle of 75°).

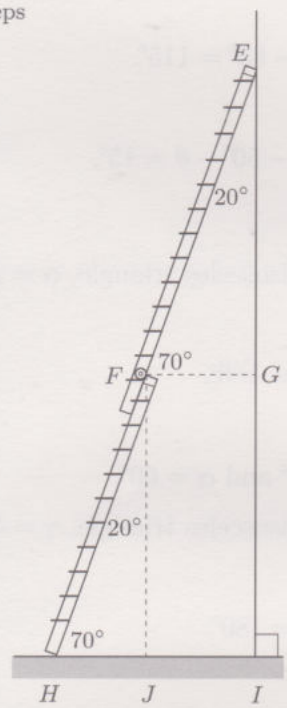
2



Domestic steps



Stair ladder



Extension ladder

There are many alternative solutions.

Here are some similar triangles which are identified by using the labels given in the diagram above:

domestic steps	$\triangle ACD$ and $\triangle ACB$,
stair ladder	$\triangle STR, \triangle STQ, \triangle PQR, \triangle SVU$,
extension ladder	$\triangle FJH, \triangle EIH, \triangle EGF$.

Some congruent triangles are

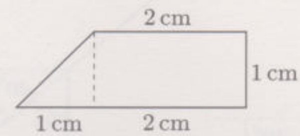
domestic steps	$\triangle ACD$ and $\triangle ACB$,
stair ladder	$\triangle QST$ and $\triangle STR$.

Section 7.3.1

1

(a) Area of triangle $= \frac{1}{2} \times \text{base} \times \text{height}$
$= \frac{1}{2} \times 8\,\text{cm} \times 6\,\text{cm}$
$= 24\,\text{cm}^2$.

(b) The trapezium can be split into a triangle and a rectangle:



Area of trapezium = area of triangle
+ area of rectangle
$= (\frac{1}{2} \times 1\,\text{cm} \times 1\,\text{cm})$
$+ (2\,\text{cm} \times 1\,\text{cm})$
$= 2.5\,\text{cm}^2$.

2

Area of front of box = area of back of box
$= 40\,\text{cm} \times 25\,\text{cm}$
$= 1000\,\text{cm}^2$.

Area of nearside = area of farside
$= 25\,\text{cm} \times 25\,\text{cm}$
$= 625\,\text{cm}^2$.

Area of top = area of base
$= 40\,\text{cm} \times 25\,\text{cm}^2$
$= 1000\,\text{cm}^2$.

Total area of box
$= (2 \times 1000\,\text{cm}^2) +$
$(2 \times 625\,\text{cm}^2) + (2 \times 1000\,\text{cm}^2)$
$= 5250\,\text{cm}^2$.

Therefore, the amount of material needed is $5250\,\text{cm}^2$.

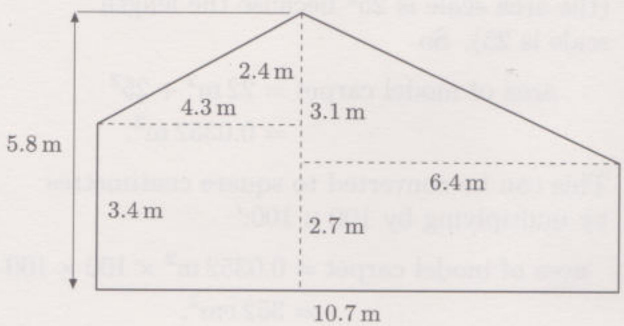
3

Area of floor $= 5\,\text{m} \times 4\,\text{m} = 20\,\text{m}^2$.
Area of rug $= 6\,\text{m}^2$.
Area to be varnished $= 20\text{m}^2 - 6\,\text{m}^2 = 14\,\text{m}^2$.
Number of tins of varnish required $= \dfrac{14}{2.5} = 5.6$.

So six tins will have to be purchased.

4

The end wall of the bungalow, minus the windows, can be divided into simple shapes as shown.



Area of left triangle $= \frac{1}{2} \times 4.3\,\text{m} \times 2.4\,\text{m} = 5.16\,\text{m}^2$.

Area of left rectangle $= 4.3\,\text{m} \times 3.4\,\text{m} = 14.62\,\text{m}^2$.

Area of right triangle $= \frac{1}{2} \times 6.4\,\text{m} \times 3.1\,\text{m} = 9.92\,\text{m}^2$.

Area of right rectangle $= 6.4\,\text{m} \times 2.7\,\text{m} = 17.28\,\text{m}^2$.

Total area of end wall = sum of areas above

$= 46.98\,\text{m}^2$.

The dimensions of the windows, in metres, are 2.2 m by 1.45 m and 1.25 m by 0.88 m, respectively.

Area of windows $= (2.2\,\text{m} \times 1.45\,\text{m}) + (1.25\,\text{m} \times 0.88\,\text{m})$

$= 3.19\,\text{m}^2 + 1.1\,\text{m}^2 = 4.29\,\text{m}^2$.

Area to be painted = total area − area of windows

$= 46.98\,\text{m}^2 - 4.29\,\text{m}^2 = 42.69\,\text{m}^2$.

5

Area of one side of sloping roof

$= 2.12\,\text{m} \times 5\,\text{m} = 10.6\,\text{m}^2$.

Area of one side of tent $= 2\,\text{m} \times 5\,\text{m} = 10\,\text{m}^2$.

Area of front/back of tent

= area of rectangle + area of triangle

$= (3\,\text{m} \times 2\,\text{m}) + (\frac{1}{2} \times 3\,\text{m} \times 1.5\,\text{m})$

$= 8.25\,\text{m}^2$.

Area of door $= 0.8\,\text{m} \times 1.75\,\text{m} = 1.4\,\text{m}^2$.

Total area of canvas

= (2 × area of one side of sloping roof)
+ (2 × area of side of tent)
+ (2 × area of front/back of tent)
− (area of door)

$= (2 \times 10.6\,\text{m}^2) + (2 \times 10\,\text{m}^2) + (2 \times 8.25\,\text{m}^2) - 1.4\,\text{m}^2$

$= 56.3\,\text{m}^2$.

So $56.3\,\text{m}^2$ of canvas are needed.

(In practice, the amount needed will depend upon the width of the canvas and on how many joins there are. It is likely that at least $60\,\text{m}^2$ will be needed.)

Section 7.3.2

1

(a)

$$\begin{aligned}\text{Area of circle} &= \pi \times (\text{radius})^2\\ &= \pi \times (8\,\text{cm})^2\\ &= 201\,\text{cm}^2 \text{ (to the nearest square centimetre).}\end{aligned}$$

(b)

$$\begin{aligned}\text{Area of circle} &= \pi \times (\text{radius})^2\\ &= \pi \times (15\,\text{m})^2\\ &= 707\,\text{m}^2 \text{(to the nearest square metre).}\end{aligned}$$

2

Surface area of central circle

$$\begin{aligned}&= \pi r^2 = \pi \times (0.4\,\text{m})^2\\ &= 0.5027\,\text{m}^2 \text{ (to 4 d.p.).}\end{aligned}$$

There are four slabs in this circle, so each slab will have

$$\text{surface area} = \frac{0.5027}{4}\text{m}^2 = 0.1257\,\text{m}^2.$$

The surface area of the eight slabs in the inner ring is calculated by subtracting the central circle from the circle with radius 0.8 m:

$$\begin{aligned}\text{surface area} &= \pi(0.8\,\text{m})^2 - \pi(0.4\,\text{m})^2\\ &= 2.0106\,\text{m}^2 - 0.5027\,\text{m}^2\\ &= 1.5079\,\text{m}^2 \text{ (to 4 d.p.).}\end{aligned}$$

There are eight slabs in the inner ring, so each slab will have

$$\text{surface area} = \frac{1.5079}{8}\text{m}^2 = 0.1885\,\text{m}^2.$$

Use a similar method to find the surface area of the outer ring of slabs:

$$\begin{aligned}\text{surface area} &= \pi(1.2\,\text{m})^2 - \pi(0.8\,\text{m})^2\\ &= \pi(4.5239 - 2.0106)\,\text{m}^2\\ &= 2.5133\,\text{m}^2.\end{aligned}$$

Each slab in the outer ring will have

$$\text{surface area} = \frac{2.5133}{16}\,\text{m}^2 = 0.1571\,\text{m}^2.$$

Type of slab	Surface area/m^2	
Central slab	0.1257	Lightest
Inner ring slab	0.1885	Heaviest
Outer ring slab	0.1571	

Assuming that the slabs are of equal thickness, and are made of the same material, the weights

of the slabs will be proportional to the surface areas. So the results show that the lightest slabs are in the central circle and the heaviest in the inner ring.

Section 7.3.3

1

(a) Cross-sectional area $= \pi \times (\text{radius})^2$
$$= \pi \times (4\,\text{cm})^2$$
$$= 50.265\,\text{cm}^2 \text{ (to 3 d.p.)}.$$

So

$$\text{volume} = 50.265\,\text{cm}^2 \times 10\,\text{cm} = 502.65\,\text{cm}^3.$$

Thus the volume is $503\,\text{cm}^3$ (to the nearest cubic centimetre).
(If you used the approximate value of 3.14 for π, you will have got a cross-sectional area of $50.24\,\text{cm}^2$ and a volume of $502.4\,\text{cm}^3$.)

(b) Cross-sectional area
$$= \text{area of square} + \text{area of triangle}$$
$$= (5\,\text{m} \times 5\,\text{m}) + (\tfrac{1}{2} \times 5\,\text{m} \times 5\,\text{m})$$
$$= 37.5\,\text{m}^2.$$

So

$$\text{volume} = 37.5\,\text{m}^2 \times 10\,\text{m} = 375\,\text{m}^3.$$

2

Car A has four cylinders, each with a radius of 4.3 cm and a height of 8.6 cm. The volume of one cylinder is calculated by using the formula

$$\text{volume of a cylinder} = \pi r^2 h.$$

So, the four cylinders will have

$$\text{total volume} = 4[\pi(4.3\,\text{cm})^2 \times 8.6\,\text{cm}]$$
$$\simeq 1998.2\,\text{cm}^3 \text{ (to 1 d.p.)}.$$

Car B has four cylinders, each with a radius of 4.6 cm and a height of 7.5 cm. From the same formula, the four cylinders will have

$$\text{total volume} = 4[\pi(4.6\,\text{cm})^2 \times 7.5\,\text{cm}]$$
$$\simeq 1994.3\,\text{cm}^3 \text{ (to 1 d.p.)}.$$

Therefore, both engines have a cubic capacity very close to $2000\,\text{cm}^3$. They are both said to have two-litre engines. Hence the claims of both manufacturers are true.

Section 7.3.4

1

(a) Since 1 cm in the model represents 25 cm in real life, areas must be scaled by 25×25 (the area scale is 25^2 because the length scale is 25). So

$$\text{area of model carpet} = 22\,\text{m}^2 \div 25^2$$
$$= 0.0352\,\text{m}^2.$$

This can be converted to square centimetres by multiplying by 100×100:

$$\text{area of model carpet} = 0.0352\,\text{m}^2 \times 100 \times 100$$
$$= 352\,\text{cm}^2.$$

Alternatively, you could convert to cm^2 first:

$$22\,\text{m}^2 = 22 \times 100 \times 100\,\text{cm}^2$$
$$= 220\,000\,\text{cm}^2.$$

So

$$\text{area of model carpet} = 220\,000\,\text{cm}^2 \div 25^2$$
$$= 352\,\text{cm}^2.$$

2

Since 1 cm in the model represents 20 cm in real life, volumes must be scaled by $20 \times 20 \times 20$.

So the volume of the tender in real life must be

$$\frac{1}{200}\,\text{m}^3 \times 20 \times 20 \times 20 = 40\,\text{m}^3.$$

Thus the volume of coal that could be carried in the real engine's tender is $40\,\text{m}^3$.

Index

Prep Res
Book B

MU120 Open Mathematics

Preparatory Resource Book A Modules 1–4
Preparatory Resource Book B Modules 5–7

MU120 Preparatory Resource Book B
SUP 00252 2